

What Core Curriculum Should School Districts be Using?	
NYS Mathematics Core Curriculum (Revised March 2005) <a href="#">PDF</a> / <a href="#">Microsoft Word</a> / <a href="#">HTML</a>	<a href="#">Mathematics Resource Guide with Core Curriculum</a> (1999)
The PreK-8 portion of <i>this</i> core is effective September 2005.	The Commencement-level portion of <i>this</i> core remains in effect until the October NYS Board of Regents meeting. Please visit this site the week of October 10, 2005.



# Mathematics

## Core Curriculum

## MST Standard 3

## Prekindergarten - Grade 12

Revised March 2005



THE UNIVERSITY OF THE STATE OF NEW YORK  
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### *Introduction*

Every teacher of mathematics, whether at the elementary, middle, or high school level, has an individual goal to provide students with the knowledge and understanding of the mathematics necessary to function in a world very dependent upon the application of mathematics. Instructionally, this goal translates into three components:

- ❖ conceptual understanding
- ❖ procedural fluency
- ❖ problem solving

***Conceptual understanding*** consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

***Procedural fluency*** is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). Although the word *procedural* may imply an arithmetic procedure to some, it also refers to being fluent with procedures from other branches of mathematics, such as measuring the size of an angle using a protractor. The use of calculators need not threaten the development of students' computational skills. On the contrary, calculators can enhance both understanding and computing if used properly and effectively. Accuracy and efficiency with procedures are important, but they should be developed through understanding. When students learn procedures through understanding, they are more likely to remember the procedures and less likely to make common computational errors.

***Problem solving*** is the ability to formulate, represent, and solve mathematical problems. Problems generally fall into three types:

- ❖ one-step problems
- ❖ multi-step problems
- ❖ process problems

Most problems that students will encounter in the real world are multi-step or process problems. Solution of these problems involves the integration of conceptual understanding and procedural knowledge. Students need to have a broad range of strategies upon which to draw. Selection of a strategy for finding the solution to a problem is often the most difficult part of the solution. Therefore, mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers. A concept or procedure in itself is not useful in problem solving unless one recognizes when and where to use it as well as when and where it does not apply. Many textbook problems are not typical of those that students will meet in real life. Therefore, students need to be able to have a general understanding of how to analyze a problem and how to choose the most useful strategy for solving the problem.

Individually, each of these components (conceptual understanding, procedural fluency, and problem solving) is necessary but not sufficient for a student to be mathematically proficient. They are not, however, independent of each other. They are integrally related, need to be taught simultaneously, and should be a component of every lesson.

The mathematics standard presented in this document states that students will:

- ❖ understand the concepts of and become proficient with the skills of mathematics;
- ❖ communicate and reason mathematically;
- ❖ become problem solvers by using appropriate tools and strategies;

through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability. Mathematics should be viewed as a whole body of knowledge, not as a set of individual components. Therefore, local mathematics curriculum, instruction, and assessment should be designed to support and sustain the components of this standard.

New York State's yearly 3-8 mathematics assessments, as required by NCLB federal legislation, will provide data measuring student progress toward obtaining mathematical proficiency. Since the state assessments will measure conceptual understanding, procedural fluency, and problem solving, local assessments should measure these components as well. Thus, many schools may need to provide teachers with significant professional staff development to assist them in developing local assessments.

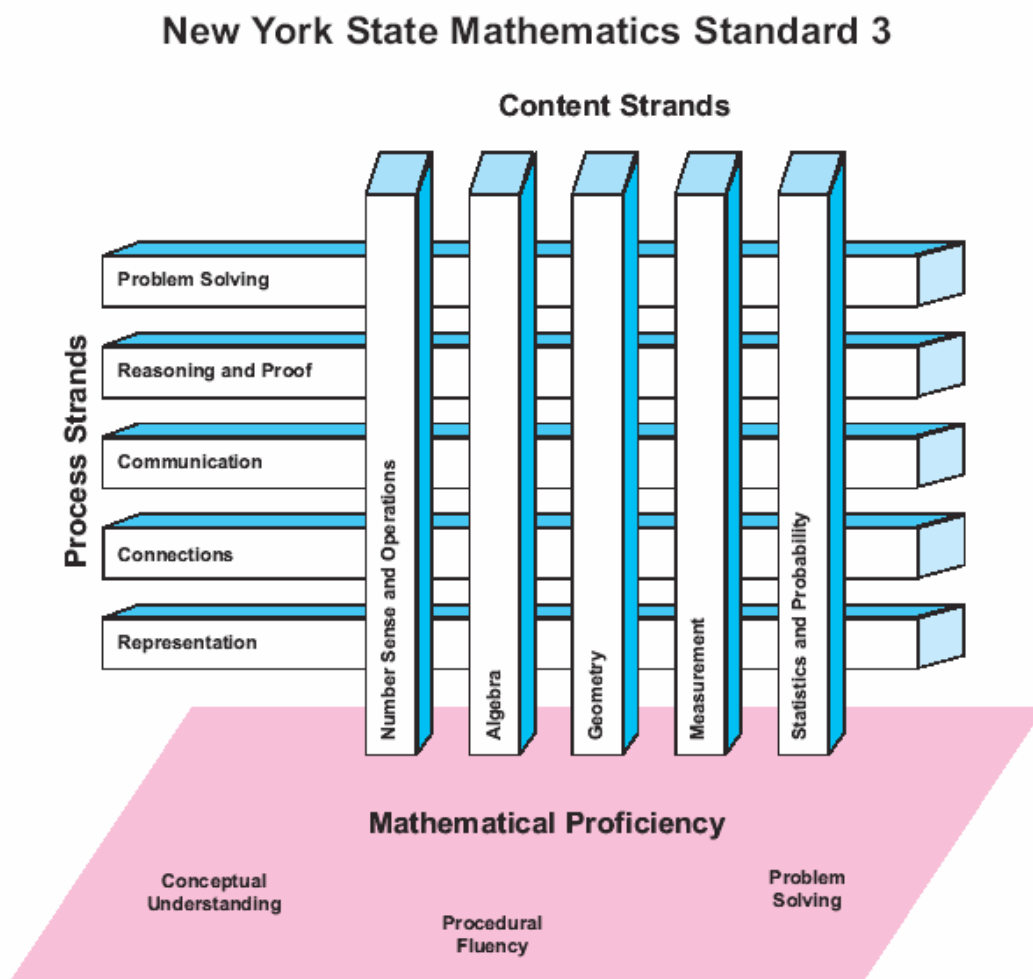
In this document conceptual understanding, procedural fluency, and problem solving are represented as *process strands* and *content strands*. These strands help to define what students should know and be able to do as a result of their engagement in the study of mathematics.

***Process Strands:*** The process strands (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) highlight ways of acquiring and using content knowledge. These process strands help to give meaning to mathematics and help students to see mathematics as a discipline rather than a set of isolated skills. Student engagement in mathematical content is accomplished through these process strands. Students will gain a better understanding of mathematics and have longer retention of mathematical knowledge as they solve problems, reason mathematically, prove mathematical relationships, participate in mathematical discourse, make mathematical connections, and model and represent mathematical ideas in a variety of ways.

***Content Strands:*** The content strands (Number Sense and Operations, Algebra, Geometry, Measurement, and Statistics and Probability) explicitly describe the content that students should learn. Each school's mathematics curriculum developed from these strands should include a broad range of content. This broad range of content, taught in an integrated fashion, allows students to see how various mathematics knowledge is related, not only within mathematics, but also to other disciplines and the real world as well. The performance indicators listed under each band within a strand are intended to assist teachers in determining what the outcomes of instruction should be. The instruction should engage students in the construction of this knowledge and should integrate conceptual understanding and problem solving with these

performance indicators. The performance indicators should not be viewed as a checklist of skills void of understanding and application.

Students will only become successful in mathematics if they see mathematics as a whole, not as isolated skills and facts. As school districts develop their own mathematics curriculum based upon the statements in this standards document, attention must be given to both content and process strands. Likewise, as teachers develop their instructional plans and their assessment techniques, they also must give attention to the integration of process and content. To do otherwise would produce students who have temporary knowledge and who are unable to apply mathematics in realistic settings. Curriculum, instruction, and assessment are intricately related and must be designed with this in mind. All three domains must address conceptual understanding, procedural fluency, and problem solving. If this is accomplished, school districts will produce students who will (1) have mathematical knowledge, (2) have an understanding of mathematical concepts, and (3) be able to apply mathematics in the solution of problems.



Adapted from *Mathematics Framework for the 1996, 2000, and 2003 National Assessment of Educational Progress*.

School districts and individual teachers should be aware that this document is a standards document that guides the development of local curriculum. Local school districts remain responsible for developing curriculum aligned to the New York State standards. In this document the mathematics standard is succinctly stated. The standard outlines what students should know and be able to do in mathematics. The content strands, consisting of bands and performance indicators within each band, and the performance indicators of the process strands help to define how the standard will be met. Each school district's mathematics curriculum should be developed to assure that all students achieve the performance indicators for both the process and content strands.

Helping all students become proficient in mathematics is an imperative goal for every school. It is the hope that this standards document will assist schools and individual teachers in meeting this goal. For additional information visit the New York State Education Department mathematics website <http://www.emsc.nysed.gov/ciai/mst/math.html> .

*Proposed Mathematics Standard, Content Strands, Process Strands, Bands within the Content Strands, and Grade-By-Grade Performance Indicators*

**Mathematics, Science, and Technology - Standard 3**

Students will:

- understand the concepts of and become proficient with the skills of mathematics;
- communicate and reason mathematically;
- become problem solvers by using appropriate tools and strategies;

through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability.

**The Five Content Strands**

**Number Sense and Operations Strand**

Students will:

- understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and procedures, and how they relate to one another;
- compute accurately and make reasonable estimates.

**Algebra Strand**

Students will:

- represent and analyze algebraically a wide variety of problem solving situations;
- perform algebraic procedures accurately;
- recognize, use, and represent algebraically patterns, relations, and functions.

**Geometry Strand**

Students will:

- use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes;
- identify and justify geometric relationships, formally and informally;
- apply transformations and symmetry to analyze problem solving situations;
- apply coordinate geometry to analyze problem solving situations.

**Measurement Strand**

Students will:

- determine what can be measured and how, using appropriate methods and formulas;
- use units to give meaning to measurements;
- understand that all measurement contains error and be able to determine its significance;
- develop strategies for estimating measurements.



### **Statistics and Probability Strand**

Students will:

- collect, organize, display, and analyze data;
- make predictions that are based upon data analysis;
- understand and apply concepts of probability.

## **The Five Process Strands**

### **Problem Solving Strand**

Students will:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

### **Reasoning and Proof Strand**

Students will:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

### **Communication Strand**

Students will:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

### **Connections Strand**

Students will:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

### **Representation Strand**

Students will:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

## **Bands Within the Content Strands**

### **Number Sense and Operations**

- Number Systems
- Number Theory
- Operations
- Estimation

### **Algebra**

- Variables and Expressions
- Equations and Inequalities
- Patterns, Relations, and Functions
- Coordinate Geometry
- Trigonometric Functions

### **Geometry**

- Shapes
- Geometric Relationships
- Transformational Geometry
- Coordinate Geometry
- Constructions
- Locus
- Informal Proofs
- Formal Proofs

### **Measurement**

- Units of Measurement
- Tools and Methods
- Units
- Error and Magnitude
- Estimation

### **Statistics and Probability**

- Collection of Data
- Organization and Display of Data
- Analysis of Data
- Predictions from Data
- Probability

## Pre-Kindergarten

### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

PK.PS.1 Explore, examine, and make observations about a social problem or mathematical situation

PK.PS.2 Interpret information correctly, identify the problem, and generate possible solutions

*Students will solve problems that arise in mathematics and in other contexts.*

PK.PS.3 Act out or model with manipulatives activities involving mathematical content from literature and/or story telling

PK.PS.4 Formulate problems and solutions from everyday situations (e.g., as counting the number of children in the class or using the calendar to teach counting)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

PK.PS.5 Use informal counting strategies to find solutions

PK.PS.6 Experience teacher-directed questioning process to understand problems

PK.PS.7 Compare and discuss ideas for solving a problem with teacher and/or students to justify their thinking

PK.PS.8 Use manipulatives (e.g., tiles, blocks) to model the action in problems

PK.PS.9 Use drawings/pictures to model the action in problems

*Students will monitor and reflect on the process of mathematical problem solving.*

PK.PS.10 Explain to others how a problem was solved, giving strategies

## **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

PK.RP.1 Understand that mathematical statements can be true or false

*Students will make and investigate mathematical conjectures.*

PK.RP.2 Investigate the use of knowledgeable guessing as a mathematical tool

PK.RP.3 Explore guesses, using a variety of objects and manipulatives

*Students will develop and evaluate mathematical arguments and proofs.*

PK.RP.4 Listen to claims other students make

## **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

PK.CM.1 Understand how to organize their thought processes with teacher guidance

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

PK.CM.2 Share mathematical ideas through the manipulation of objects, drawings, pictures, and verbal explanations

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

PK.CM.3 Listen to solutions shared by other students

PK.CM.4 Formulate mathematically relevant questions with teacher guidance

*Students will use the language of mathematics to express mathematical ideas precisely.*

PK.CM.5 Use appropriate mathematical terms, vocabulary, and language

## **Connections Strand**

*Students will recognize and apply mathematics in contexts outside of mathematics.*

PK.CN.1 Recognize the presence of mathematics in their daily lives

PK.CN.2 Use counting strategies to solve problems in their daily lives

PK.CN.3 Recognize and apply mathematics to objects and pictures

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

PK.R.1 Use multiple representations, including verbal language, acting out or modeling a situation, and drawing pictures as representations

PK.R.2 Use standard and nonstandard representations

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

PK.R.3 Use objects to show and understand physical phenomena (e.g., guess the number of cookies in a package)

PK.R.4 Use objects to show and understand social phenomena (e.g., count and represent sharing cookies between friends)

PK.R.5 Use objects to show and understand mathematical phenomena (e.g., draw pictures to show a story problem, show number value using fingers on your hand)

## **Number Sense and Operations Strand**

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

*Number Systems*

PK.N.1 Count the items in a collection and know the last counting word tells how many items are in the collection (1 to 10)

PK.N.2 Count out (produce) a collection of a specified size 1 to 10

PK.N.3 Verbally count by 1's to 10

PK.N.4 Explore the different representations of a group of objects

PK.N.5 Draw pictures or other informal symbols to represent a spoken number up to 5

PK.N.6 Draw pictures or other informal symbols to represent how many in a collection up to 5

PK.N.7 Recognize numerals (0-5)

PK.N.8 Use and understand the terms *first* and *last*

*Students will understand meanings of operations and procedures, and how they relate to one another.*

*Operations* PK.N.9 Develop addition and subtraction readiness with sums up to 4 and subtraction involving one to four items, using manipulatives

### **Algebra Strand**

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

*Patterns, Relations, and Functions* PK.A.1 Duplicate simple patterns using concrete objects

### **Geometry Strand**

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

*Shapes* PK.G.1 Match shapes, first with same size and orientation, then with different sizes and orientation

PK.G.2 Informally play with solids (e.g., building blocks)

### **Measurement Strand**

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of Measurement* PK.M.1 Develop language such as bigger, longer, and taller to discuss length

PK.M.2 Relate specific times such as day and night

## Statistics and Probability Strand

*Students will collect, organize, display, and analyze data.*

- |   |   |
|---|---|
| <i>Organization and Display of Data</i> | PK.S.1 Sort and organize objects by one attribute (e.g., color, size, or shape) |
|   | PK.S.2 Use physical objects to make graphs                                      |
| <i>Analysis of Data</i>                 | PK.S.3 Count and compare groups formed (quantify groups formed)                 |
|   | PK.S.4 Describe the attributes of objects                                       |

### Kindergarten

## Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

- K.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
- K.PS.2 Interpret information correctly, identify the problem, and generate possible solutions

*Students will solve problems that arise in mathematics and in other contexts.*

- K.PS.3 Act out or model with manipulatives activities involving mathematical content from literature and/or story telling
- K.PS.4 Formulate problems and solutions from everyday situations (e.g., counting the number of children in the class, using the calendar to teach counting).

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- K.PS.5 Use informal counting strategies to find solutions
- K.PS.6 Experience teacher-directed questioning process to understand

problems

K.PS.7 Compare and discuss ideas for solving a problem with teacher and/or students to justify their thinking

K.PS.8 Use manipulatives (e.g., tiles, blocks) to model the action in problems

K.PS.9 Use drawings/pictures to model the action in problems

*Students will monitor and reflect on the process of mathematical problem solving.*

K.PS.10 Explain to others how a problem was solved, giving strategies

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

K.RP.1 Understand that mathematical statements can be true or false

*Students will make and investigate mathematical conjectures.*

K.RP.2 Investigate the use of knowledgeable guessing as a mathematical tool

K.RP.3 Explore guesses, using a variety of objects and manipulatives

*Students will develop and evaluate mathematical arguments and proofs.*

K.RP.4 Listen to claims other students make

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

K.CM.1 Understand how to organize their thought processes with teacher guidance

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

K.CM.2 Share mathematical ideas through the manipulation of objects, drawings, pictures, and verbal explanations



*Students will analyze and evaluate the mathematical thinking and strategies of others.*

K.CM.3 Listen to solutions shared by other students

K.CM.4 Formulate mathematically relevant questions with teacher guidance

*Students will use the language of mathematics to express mathematical ideas precisely.*

K.CM.5 Use appropriate mathematical terms, vocabulary, and language

### **Connections Strand**

*Students will recognize and apply mathematics in contexts outside of mathematics.*

K.CN.1 Recognize the presence of mathematics in their daily lives

K.CN.2 Use counting strategies to solve problems in their daily lives

K.CN.3 Recognize and apply mathematics to objects and pictures

### **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

K.R.1 Use multiple representations, including verbal language, acting out or modeling a situation, and drawing pictures as representations

K.R.2 Use standard and nonstandard representations

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

K.R.3 Use objects to show and understand physical phenomena (e.g., guess the number of cookies in a package)

K.R.4 Use objects to show and understand social phenomena (e.g., count and represent sharing cookies between friends)

K.R.5 Use objects to show and understand mathematical phenomena (e.g., draw pictures to show a story problem, show number

value using fingers on your hand)

## Number Sense and Operations Strand

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

### *Number Systems*

- K.N.1 Count the items in a collection and know the last counting word tells how many items are in the collection (1 to 10)
- K.N.2 Count out (produce) a collection of a specified size 1 to 10
- K.N.3 Numerically label a data set of 1 to 5
- K.N.4 Verbally count by 1's to 20
- K.N.5 Verbally count backwards from 10
- K.N.6 Represent collections with a finger pattern up to 10
- K.N.7 Draw pictures or other informal symbols to represent a spoken number up to 10
- K.N.8 Draw pictures or other informal symbols to represent how many in a collection up to 10
- K.N.9 Write numbers 1-10 to represent a collection
- K.N.10 Visually determine how many more or less, and then using the verbal counting sequence, match and count 1-10
- K.N.11 Use and understand verbal ordinal terms, first to tenth

*Students will understand meanings of operations and procedures, and how they relate to one another.*

### *Operations*

- K.N.12 Solve and create addition and subtraction verbal word problems (use counting-based strategies, such as counting on and to ten)
- K.N.13 Determine sums and differences by various means

## Algebra Strand

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

- Patterns, Relations,  
and Functions*
- K.A.1 Use a variety of manipulatives to create patterns using attributes of color, size, or shape
- K.A.2 Recognize, describe, extend, and create patterns that repeat (e.g., ABABAB or ABAABAAAB)

## **Geometry Strand**

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

- Shapes*
- K.G.1 Describe characteristics and relationships of geometric objects

*Students will identify and justify geometric relationships, formally and informally.*

- Geometric Relationships*
- K.G.2 Sort groups of objects by size and size order (increasing and decreasing)

*Students will apply transformations and symmetry to analyze problem solving situations.*

- Transformational Geometry*
- K.G.3 Explore vertical and horizontal orientation of objects
- K.G.4 Manipulate two- and three-dimensional shapes to explore symmetry

*Students will apply coordinate geometry to analyze problem solving situations.*

- Coordinate Geometry*
- K.G.5 Understand and use ideas such as over, under, above, below, on, beside, next to, and between

## **Measurement Strand**

*Students will determine what can be measured and how, using appropriate methods and formulas.*

- Units of Measurement*
- K.M.1 Name, discuss, and compare attributes of length (longer than, shorter than)
- K.M.2 Compare the length of two objects by representing each length with string or a paper strip

K.M.3 Relate specific times such as morning, noon, afternoon, and evening to activities and absence or presence of daylight

### **Statistics and Probability Strand**

*Students will collect, organize, display, and analyze data.*

- |   |   |
|---|---|
| <i>Collection of Data</i>               | K.S.1 Gather data in response to questions posed by the teacher and students                  |
| <i>Organization and Display of Data</i> | K.S.2 Help to make simple pictographs for quantities up to 10, where one picture represents 1 |
|   | K.S.3 Sort and organize objects by two attributes (e.g., color, size, or shape)               |
|   | K.S.4 Represent data using manipulatives  |
| <i>Analysis of Data</i>                 | K.S.5 Identify more, less, and same amounts from pictographs or concrete models               |

## **Grade 1**

### **Problem Solving Strand**

*Students will build new mathematical knowledge through problem solving.*

- 1.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
- 1.PS.2 Interpret information correctly, identify the problem, and generate possible solutions

*Students will solve problems that arise in mathematics and in other contexts.*

- 1.PS.3 Act out or model with manipulatives activities involving mathematical content from literature and/or story telling

- 1.PS.4 Formulate problems and solutions from everyday situations (e.g., counting the number of children in the class or using the calendar to teach counting)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 1.PS.5 Use informal counting strategies to find solutions
- 1.PS.6 Experience teacher-directed questioning process to understand problems
- 1.PS.7 Compare and discuss ideas for solving a problem with teacher and/or students to justify their thinking
- 1.PS.8 Use manipulatives (e.g., tiles, blocks) to model the action in problems
- 1.PS.9 Use drawings/pictures to model the action in problems

*Students will monitor and reflect on the process of mathematical problem solving.*

- 1.PS.10 Explain to others how a problem was solved, giving strategies and justifications

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- 1.RP.1 Understand that mathematical statements can be true or false
- 1.RP.2 Recognize that mathematical ideas need to be supported by evidence

*Students will make and investigate mathematical conjectures.*

- 1.RP.3 Investigate the use of knowledgeable guessing as a mathematical tool
- 1.RP.4 Explore guesses, using a variety of objects and manipulatives

*Students will develop and evaluate mathematical arguments and proofs.*

- 1.RP.5 Justify general claims, using manipulatives
- 1.RP.6 Develop and explain an argument verbally or with objects

1.RP.7 Listen to and discuss claims other students make

*Students will select and use various types of reasoning and methods of proof.*

1.RP.8 Use trial and error strategies to verify claims

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

1.CM.1 Understand how to organize their thought processes with teacher guidance

1.CM.2 Verbally support their reasoning and answer

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

1.CM.3 Share mathematical ideas through the manipulation of objects, drawings, pictures, charts, and symbols in both written and verbal explanations

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

1.CM.4 Listen to solutions shared by other students

1.CM.5 Formulate mathematically relevant questions

*Students will use the language of mathematics to express mathematical ideas precisely.*

1.CM.6 Use appropriate mathematical terms, vocabulary, and language

### **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

1.CN.1 Recognize the connections of patterns in their everyday experiences to mathematical ideas

1.CN.2 Understand the connections between numbers and the quantities they represent

1.CN.3 Compare the similarities and differences of mathematical ideas

*Students will understand how mathematical ideas interconnect and build on one another to*

*produce a coherent whole.*

- 1.CN.4 Understand how models of situations involving objects, pictures, and symbols relate to mathematical ideas
- 1.CN.5 Understand meanings of operations and how they relate to one another
- 1.CN.6 Understand how mathematical models represent quantitative relationships

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- 1.CN.7 Recognize the presence of mathematics in their daily lives
- 1.CN.8 Recognize and apply mathematics to solve problems
- 1.CN.9 Recognize and apply mathematics to objects, pictures, and symbols

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 1.R.1 Use multiple representations including verbal and written language, acting out or modeling a situation, drawings, and/or symbols as representations
- 1.R.2 Share mental images of mathematical ideas and understandings
- 1.R.3 Use standard and nonstandard representations

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 1.R.4 Connect mathematical representations with problem solving

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 1.R.5 Use mathematics to show and understand physical phenomena (e.g., estimate and represent the number of apples in a tree)
- 1.R.6 Use mathematics to show and understand social phenomena

(e.g., count and represent sharing cookies between friends)

- 1.R.7 Use mathematics to show and understand mathematical phenomena (e.g., draw pictures to show a story problem, show number value using fingers on your hand)

## Number Sense and Operations Strand

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

### *Number Systems*

- 1.N.1 Count the items in a collection and know the last counting word tells how many items are in the collection (1 to 100)
- 1.N.2 Count out (produce) a collection of a specified size (10 to 100 items), using groups of ten
- 1.N.3 Quickly see and label with a number, collections of 1 to 10
- 1.N.4 Count by 1's to 100
- 1.N.5 Skip count by 10's to 100
- 1.N.6 Skip count by 5's to 50
- 1.N.7 Skip count by 2's to 20
- 1.N.8 Verbally count from a number other than one by 1's
- 1.N.9 Count backwards from 20 by 1's
- 1.N.10 Draw pictures or other informal symbols to represent a spoken number up to 20
- 1.N.11 Identify that spacing of the same number of objects does not affect the quantity (conservation)
- 1.N.12 Arrange objects in size order (increasing and decreasing)
- 1.N.13 Write numbers to 100
- 1.N.14 Read the number words *one, two, three...ten*
- 1.N.15 Explore and use place value



1.N.16 Compare and order whole numbers up to 100

1.N.17 Develop an initial understanding of the base ten system:

10 ones = 1 ten

10 tens = 1 hundred

1.N.18 Use a variety of strategies to compose and decompose one-digit numbers

1.N.19 Understand the commutative property of addition

1.N.20 Name the number before and the number after a given number, and name the number(s) between two given numbers up to 100 (with and without the use of a number line or a hundreds chart)

1.N.21 Use before, after, or between to order numbers to 100 (with or without the use of a number line)

1.N.22 Use the words higher, lower, greater, and less to compare two numbers

1.N.23 Use and understand verbal ordinal terms, first to twentieth

***Students will understand meanings of operations and procedures, and how they relate to one another.***

*Operations*

1.N.24 Develop and use strategies to solve addition and subtraction word problems

1.N.25 Represent addition and subtraction word problems and their solutions as number sentences

1.N.26 Create problem situations that represent a given number sentence

1.N.27 Use a variety of strategies to solve addition and subtraction problems with one- and two-digit numbers without regrouping

1.N.28 Demonstrate fluency and apply addition and subtraction facts to and including 10

1.N.29 Understand that different parts can be added to get the same whole

***Students will compute accurately and make reasonable estimates.***

*Estimation*

1.N.30 Estimate the number in a collection to 50 and then compare by counting the actual items in the collection

## Algebra Strand

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

*Patterns, Relations, and Functions*    1.A.1 Determine and discuss patterns in arithmetic (what comes next in a repeating pattern, using numbers or objects)

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

*Shapes*                                    1.G.1 Match shapes and parts of shapes to justify congruency

1.G.2 Recognize, name, describe, create, sort, and compare two-dimensional and three-dimensional shapes

*Students will apply transformations and symmetry to analyze problem solving situations.*

*Transformational Geometry*    1.G.3 Experiment with slides, flips, and turns of two-dimensional shapes

1.G.4 Identify symmetry in two-dimensional shapes

*Students will apply coordinate geometry to analyze problem solving situations.*

*Coordinate Geometry*            1.G.5 Recognize geometric shapes and structures in the environment

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of Measurement*            1.M.1 Recognize length as an attribute that can be measured

1.M.2 Use non-standard units (including finger lengths, paper clips, students' feet and paces) to measure both vertical and horizontal lengths

1.M.3 Informally explore the standard unit of measure, inch

*Students will use units to give meaning to measurements.*

- Units*
- 1.M.4 Know vocabulary and recognize coins (penny, nickel, dime, quarter)
  - 1.M.5 Recognize the cent notation as ¢
  - 1.M.6 Use different combinations of coins to make money amounts up to 25 cents
  - 1.M.7 Recognize specific times (morning, noon, afternoon, evening)
  - 1.M.8 Tell time to the hour, using both digital and analog clocks
  - 1.M.9 Know the days of the week and months of the year in sequence
  - 1.M.10 Classify months and connect to seasons and other events

*Students will develop strategies for estimating measurements.*

- Estimation*
- 1.M.11 Select and use non-standard units to estimate measurements

## **Statistics and Probability Strand**

*Students will collect, organize, display, and analyze data.*

- Collection of Data*
- 1.S.1 Pose questions about themselves and their surroundings
  - 1.S.2 Collect and record data related to a question
- Organization and Display of Data*
- 1.S.3 Display data in simple pictographs for quantities up to 20 with units of one
  - 1.S.4 Display data in bar graphs using concrete objects with intervals of one
  - 1.S.5 Use Venn diagrams to sort and describe data
- Analysis of Data*
- 1.S.6 Interpret data in terms of the words: most, least, greater than, less than, or equal to
  - 1.S.7 Answer simple questions related to data displayed in pictographs (e.g., category with most, how many more in a category compared to another, how many all together in two categories)

*Students will make predictions that are based upon data analysis.*

- Predictions from Data*
- 1.S.8 Discuss conclusions and make predictions in terms of the words likely and unlikely
  - 1.S.9 Construct a question that can be answered by using information from a graph

<h2>Grade 2</h2>
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### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

- 2.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
- 2.PS.2 Interpret information correctly, identify the problem, and generate possible solutions

*Students will solve problems that arise in mathematics and in other contexts.*

- 2.PS.3 Act out or model with manipulatives activities involving mathematical content from literature and/or story telling
- 2.PS.4 Formulate problems and solutions from everyday situations (e.g., counting the number of children in the class, using the calendar to teach counting).

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 2.PS.5 Use informal counting strategies to find solutions
- 2.PS.6 Experience teacher-directed questioning process to understand problems
- 2.PS.7 Compare and discuss ideas for solving a problem with teacher and/or students to justify their thinking

2.PS.8 Use manipulatives (e.g., tiles, blocks) to model the action in problems

2.PS.9 Use drawings/pictures to model the action in problems

*Students will monitor and reflect on the process of mathematical problem solving.*

2.PS.10 Explain to others how a problem was solved, giving strategies and justifications

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

2.RP.1 Understand that mathematical statements can be true or false

2.RP.2 Recognize that mathematical ideas need to be supported by evidence

*Students will make and investigate mathematical conjectures.*

2.RP.3 Investigate the use of knowledgeable guessing as a mathematical tool

2.RP.4 Explore guesses, using a variety of objects and manipulatives

*Students will develop and evaluate mathematical arguments and proofs.*

2.RP.5 Justify general claims, using manipulatives

2.RP.6 Develop and explain an argument verbally or with objects

2.RP.7 Listen to and discuss claims other students make

*Students will select and use various types of reasoning and methods of proof.*

2.RP.8 Use trial and error strategies to verify claims

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

2.CM.1 Understand how to organize their thought processes

2.CM.2 Verbally support their reasoning and answer

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

2.CM.3 Share mathematical ideas through the manipulation of objects, drawings, pictures, charts, and symbols in both written and verbal explanations

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

2.CM.4 Listen to solutions shared by other students

2.CM.5 Formulate mathematically relevant questions

*Students will use the language of mathematics to express mathematical ideas precisely.*

2.CM.6 Use appropriate mathematical terms, vocabulary, and language

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

2.CN.1 Recognize the connections of patterns in their everyday experiences to mathematical ideas

2.CN.2 Understand and use the connections between numbers and the quantities they represent to solve problems

2.CN.3 Compare the similarities and differences of mathematical ideas

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

2.CN.4 Understand how models of situations involving objects, pictures, and symbols relate to mathematical ideas

2.CN.5 Understand meanings of operations and how they relate to one another

2.CN.6 Understand how mathematical models represent quantitative relationships

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- 2.CN.7 Recognize the presence of mathematics in their daily lives
- 2.CN.8 Recognize and apply mathematics to solve problems
- 2.CN.9 Recognize and apply mathematics to objects, pictures and symbols

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 2.R.1 Use multiple representations, including verbal and written language, acting out or modeling a situation, drawings, and/or symbols as representations
- 2.R.2 Share mental images of mathematical ideas and understandings
- 2.R.3 Use standard and nonstandard representations

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 2.R.4 Connect mathematical representations with problem solving

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 2.R.5 Use mathematics to show and understand physical phenomena (e.g., estimate and represent the number of apples in a tree)
- 2.R.6 Use mathematics to show and understand social phenomena (e.g., count and represent sharing cookies between friends)
- 2.R.7 Use mathematics to show and understand mathematical phenomena (e.g., draw pictures to show a story problem or show number value using fingers on your hand)

## **Number Sense and Operations Strand**

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

- Number Systems*
- 2.N.1 Skip count to 100 by 2's, 5's, 10's
  - 2.N.2 Count back from 100 by 1's, 5's, 10's using a number chart
  - 2.N.3 Skip count by 3's to 36 for multiplication readiness
  - 2.N.4 Skip count by 4's to 48 for multiplication readiness
  - 2.N.5 Compare and order numbers to 100
  - 2.N.6 Develop an understanding of the base ten system:
    - 10 ones = 1 ten
    - 10 tens = 1 hundred
    - 10 hundreds = 1 thousand
  - 2.N.7 Use a variety of strategies to compose and decompose two-digit numbers
  - 2.N.8 Understand and use the commutative property of addition
  - 2.N.9 Name the number before and the number after a given number, and name the number(s) between two given numbers up to 100 (with and without the use of a number line or a hundreds chart)
  - 2.N.10 Use and understand verbal ordinal terms
  - 2.N.11 Read written ordinal terms (first through ninth) and use them to represent ordinal relations
  - 2.N.12 Use zero as the identity element for addition
  - 2.N.13 Recognize the meaning of zero in the place value system (0-100)
- Number Theory*
- 2.N.14 Use concrete materials to justify a number as odd or even

***Students will understand meanings of operations and procedures, and how they relate to one another.***

- Operations*
- 2.N.15 Determine sums and differences of number sentences by various means (e.g., families, related facts, inverse operations, addition doubles, and doubles plus one)
  - 2.N.16 Use a variety of strategies to solve addition and subtraction problems using one- and two-digit numbers with and without regrouping



2.N.17 Demonstrate fluency and apply addition and subtraction facts up to and including 18

2.N.18 Use doubling to add 2-digit numbers

2.N.19 Use compensation to add 2-digit numbers

2.N.20 Develop readiness for multiplication by using repeated addition

2.N.21 Develop readiness for division by using repeated subtraction, dividing objects into groups (fair share)

*Students will compute accurately and make reasonable estimates.*

*Estimation*                      2.N.22 Estimate the number in a collection to 100 and then compare by counting the actual items in the collection

## Algebra Strand

*Students will perform algebraic procedures accurately.*

*Equations and Inequalities*                      2.A.1 Use the symbols  $<$ ,  $>$ ,  $=$  (with and without the use of a number line) to compare whole numbers up to 100

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

*Patterns, Relations, and Functions*                      2.A.2 Describe and extend increasing or decreasing (+,-) sequences and patterns (numbers or objects up to 100)

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

*Shapes*                      2.G.1 Experiment with slides, flips, and turns to compare two-dimensional shapes

2.G.2 Identify and appropriately name two-dimensional shapes: circle, square, rectangle, and triangle (both regular and irregular)

2.G.3 Compose (put together) and decompose (break apart) two-dimensional shapes

*Students will identify and justify geometric relationships, formally and informally.*

*Geometric Relationships*            2.G.4 Group objects by like properties

*Students will apply transformations and symmetry to analyze problem solving situations.*

*Transformational Geometry*    2.G.5 Explore and predict the outcome of slides, flips, and turns of two-dimensional shapes

2.G.6 Explore line symmetry

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of Measurement*            2.M.1 Use non-standard and standard units to measure both vertical and horizontal lengths

2.M.2 Use a ruler to measure standard units (including whole inches and whole feet)

2.M.3 Compare and order objects according to the attribute of length

2.M.4 Recognize mass as a qualitative measure (e.g., Which is heavier? Which is lighter?)

2.M.5 Compare and order objects, using lighter than and heavier than

*Students will use units to give meaning to measurements.*

*Units*            2.M.6 Know and recognize coins (penny, nickel, dime, quarter) and bills (\$1, \$5, \$10, and \$20)

2.M.7 Recognize the whole dollar notation as \$1, etc.

2.M.8 Identify equivalent combinations to make one dollar

2.M.9 Tell time to the half hour and five minutes using both digital and analog clocks

*Students will develop strategies for estimating measurements.*

*Estimation*            2.M.10 Select and use standard (customary) and non-standard units to estimate measurements

## Statistics and Probability Strand

*Students will collect, organize, display, and analyze data.*

*Collection of Data*    2.S.1    Formulate questions about themselves and their surroundings

2.S.2    Collect and record data (using tallies) related to the question

*Organization and Display of Data*    2.S.3    Display data in pictographs and bar graphs using concrete objects or a representation of the object

*Analysis of Data*    2.S.4    Compare and interpret data in terms of describing quantity (similarity or differences)

*Students will make predictions that are based upon data analysis.*

*Predictions from Data*    2.S.5    Discuss conclusions and make predictions from graphs

# Grade 3

## Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

3.PS.1    Explore, examine, and make observations about a social problem or mathematical situation

3.PS.2    Understand that some ways of representing a problem are more helpful than others

3.PS.3    Interpret information correctly, identify the problem, and generate possible solutions

*Students will solve problems that arise in mathematics and in other contexts.*

3.PS.4    Act out or model with manipulatives activities involving mathematical content from literature

- 3.PS.5 Formulate problems and solutions from everyday situations
- 3.PS.6 Translate from a picture/diagram to a numeric expression
- 3.PS.7 Represent problem situations in oral, written, concrete, pictorial, and graphical forms
- 3.PS.8 Select an appropriate representation of a problem

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 3.PS.9 Use trial and error to solve problems
- 3.PS.10 Use process of elimination to solve problems
- 3.PS.11 Make pictures/diagrams of problems
- 3.PS.12 Use physical objects to model problems
- 3.PS.13 Work in collaboration with others to solve problems
- 3.PS.14 Make organized lists to solve numerical problems
- 3.PS.15 Make charts to solve numerical problems
- 3.PS.16 Analyze problems by identifying relationships
- 3.PS.17 Analyze problems by identifying relevant versus irrelevant information
- 3.PS.18 Analyze problems by observing patterns
- 3.PS.19 State a problem in their own words

*Students will monitor and reflect on the process of mathematical problem solving.*

- 3.PS.20 Determine what information is needed to solve a problem
- 3.PS.21 Discuss with peers to understand a problem situation
- 3.PS.22 Discuss the efficiency of different representations of a problem
- 3.PS.23 Verify results of a problem
- 3.PS.24 Recognize invalid approaches

- 3.PS.25 Determine whether a solution is reasonable in the context of the original problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- 3.RP.1 Use representations to support mathematical ideas
- 3.RP.2 Determine whether a mathematical statement is true or false and explain why

*Students will make and investigate mathematical conjectures.*

- 3.RP.3 Investigate the use of knowledgeable guessing by generalizing mathematical ideas
- 3.RP.4 Make conjectures from a variety of representations

*Students will develop and evaluate mathematical arguments and proofs.*

- 3.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions
- 3.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms
- 3.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

*Students will select and use various types of reasoning and methods of proof.*

- 3.RP.8 Support an argument by trying many cases

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- 3.CM.1 Understand and explain how to organize their thought process
- 3.CM.2 Verbally explain their rationale for strategy selection
- 3.CM.3 Provide reasoning both in written and verbal form

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- 3.CM.4 Organize and accurately label work
- 3.CM.5 Share organized mathematical ideas through the manipulation of objects, drawings, pictures, charts, graphs, tables, diagrams, models, symbols, and expressions in written and verbal form
- 3.CM.6 Answer clarifying questions from others

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- 3.CM.7 Listen for understanding of mathematical solutions shared by other students
- 3.CM.8 Consider strategies used and solutions found in relation to their own work

*Students will use the language of mathematics to express mathematical ideas precisely.*

- 3.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 3.CM.10 Describe objects, relationships, solutions and rationale using appropriate vocabulary
- 3.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- 3.CN.1 Recognize, understand, and make connections in their everyday experiences to mathematical ideas
- 3.CN.2 Compare and contrast mathematical ideas
- 3.CN.3 Connect and apply mathematical information to solve problems

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

3.CN.4 Understand multiple representations and how they are related

3.CN.5 Model situations with objects and representations and be able to make observations

*Students will recognize and apply mathematics in contexts outside of mathematics.*

3.CN.6 Recognize the presence of mathematics in their daily lives

3.CN.7 Apply mathematics to solve problems that develop outside of mathematics

3.CN.8 Recognize and apply mathematics to other disciplines

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

3.R.1 Use verbal and written language, physical models, drawing charts, graphs, tables, symbols, and equations as representations

3.R.2 Share mental images of mathematical ideas and understandings

3.R.3 Recognize and use external mathematical representations

3.R.4 Use standard and nonstandard representations with accuracy and detail

*Students will select, apply, and translate among mathematical representations to solve problems.*

3.R.5 Understand similarities and differences in representations

3.R.6 Connect mathematical representations with problem solving

3.R.7 Construct effective representations to solve problems

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

3.R.8 Use mathematics to show and understand physical phenomena (e.g., estimate and represent the number of apples in a tree)

3.R.9 Use mathematics to show and understand social phenomena

(e.g., determine the number of buses required for a field trip)

- 3.R.10 Use mathematics to show and understand mathematical phenomena (e.g., use a multiplication grid to solve odd and even number problems)

## Number Sense and Operations Strand

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

### *Number Systems*

- 3.N.1 Skip count by 25's, 50's, 100's to 1,000
- 3.N.2 Read and write whole numbers to 1,000
- 3.N.3 Compare and order numbers to 1,000
- 3.N.4 Understand the place value structure of the base ten number system:  
10 ones = 1 ten  
10 tens = 1 hundred  
10 hundreds = 1 thousand
- 3.N.5 Use a variety of strategies to compose and decompose three-digit numbers
- 3.N.6 Use and explain the commutative property of addition and multiplication
- 3.N.7 Use 1 as the identity element for multiplication
- 3.N.8 Use the zero property of multiplication
- 3.N.9 Understand and use the associative property of addition
- 3.N.10 Develop an understanding of fractions as part of a whole unit and as parts of a collection
- 3.N.11 Use manipulatives, visual models, and illustrations to name and represent unit fractions ( $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ , and  $\frac{1}{10}$ ) as part of a whole or a set of objects
- 3.N.12 Understand and recognize the meaning of numerator and denominator in the symbolic form of a fraction



3.N.13 Recognize fractional numbers as equal parts of a whole

3.N.14 Explore equivalent fractions ( $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ )

3.N.15 Compare and order unit fractions ( $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ) and find their approximate locations on a number line

*Number Theory*

3.N.16 Identify odd and even numbers

3.N.17 Develop an understanding of the properties of odd/even numbers as a result of addition or subtraction

***Students will understand meanings of operations and procedures, and how they relate to one another.***

*Operations*

3.N.18 Use a variety of strategies to add and subtract 3-digit numbers (with and without regrouping)

3.N.19 Develop fluency with single-digit multiplication facts

3.N.20 Use a variety of strategies to solve multiplication problems with factors up to  $12 \times 12$

3.N.21 Use the area model, tables, patterns, arrays, and doubling to provide meaning for multiplication

3.N.22 Demonstrate fluency and apply single-digit division facts

3.N.23 Use tables, patterns, halving, and manipulatives to provide meaning for division

3.N.24 Develop strategies for selecting the appropriate computational and operational method in problem solving situations

***Students will compute accurately and make reasonable estimates.***

*Estimation*

3.N.25 Estimate numbers up to 500

3.N.26 Recognize real world situations in which an estimate (rounding) is more appropriate

3.N.27 Check reasonableness of an answer by using estimation

## Algebra Strand

*Students will perform algebraic procedures accurately.*

*Equations and Inequalities*      3.A.1 Use the symbols  $<$ ,  $>$ ,  $=$  (with and without the use of a number line) to compare whole numbers and unit fractions  
 $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \text{ and } \frac{1}{10}\right)$

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

*Patterns, Relations, and Functions*      3.A.2 Describe and extend numeric (+, -) and geometric patterns

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

*Shapes*      3.G.1 Define and use correct terminology when referring to shapes (circle, triangle, square, rectangle, rhombus, trapezoid, and hexagon)

3.G.2 Identify congruent and similar figures

3.G.3 Name, describe, compare, and sort three-dimensional shapes: cube, cylinder, sphere, prism, and cone

3.G.4 Identify the faces on a three-dimensional shape as two-dimensional shapes

*Students will apply transformations and symmetry to analyze problem solving situations.*

*Transformational Geometry*      3.G.5 Identify and construct lines of symmetry

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of Measurement*      3.M.1 Select tools and units (customary) appropriate for the length measured

- 3.M.2 Use a ruler/yardstick to measure to the nearest standard unit (whole and  $\frac{1}{2}$  inches, whole feet, and whole yards)
- 3.M.3 Measure objects, using ounces and pounds
- 3.M.4 Recognize capacity as an attribute that can be measured
- 3.M.5 Compare capacities (e.g., Which contains more? Which contains less?)
- 3.M.6 Measure capacity, using cups, pints, quarts, and gallons

*Students will use units to give meaning to measurements.*

- Units*
  - 3.M.7 Count and represent combined coins and dollars, using currency symbols (\$0.00)
  - 3.M.8 Relate unit fractions to the face of the clock:
    - Whole = 60 minutes
    - $\frac{1}{2}$  = 30 minutes
    - $\frac{1}{4}$  = 15 minutes

*Students will develop strategies for estimating measurements.*

- Estimation*
  - 3.M.9 Tell time to the minute, using digital and analog clocks
  - 3.M.10 Select and use standard (customary) and non-standard units to estimate measurements

## **Statistics and Probability Strand**

*Students will collect, organize, display, and analyze data.*

- Collection of Data*
  - 3.S.1 Formulate questions about themselves and their surroundings
  - 3.S.2 Collect data using observation and surveys, and record appropriately
- Organization and Display of Data*
  - 3.S.3 Construct a frequency table to represent a collection of data
  - 3.S.4 Identify the parts of pictographs and bar graphs
  - 3.S.5 Display data in pictographs and bar graphs
  - 3.S.6 State the relationships between pictographs and bar graphs

*Analysis of Data*      3.S.7    Read and interpret data in bar graphs and pictographs

*Students will make predictions that are based upon data analysis.*

*Predictions from Data*      3.S.8    Formulate conclusions and make predictions from graphs

## Grade 4

### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

- 4.PS.1    Explore, examine, and make observations about a social problem or mathematical situation
- 4.PS.2    Understand that some ways of representing a problem are more helpful than others
- 4.PS.3    Interpret information correctly, identify the problem, and generate possible solutions

*Students will solve problems that arise in mathematics and in other contexts.*

- 4.PS.4    Act out or model with manipulatives activities involving mathematical content from literature
- 4.PS.5    Formulate problems and solutions from everyday situations
- 4.PS.6    Translate from a picture/diagram to a numeric expression
- 4.PS.7    Represent problem situations in oral, written, concrete, pictorial, and graphical forms
- 4.PS.8    Select an appropriate representation of a problem

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 4.PS.9    Use trial and error to solve problems

- 4.PS.10 Use process of elimination to solve problems
- 4.PS.11 Make pictures/diagrams of problems
- 4.PS.12 Use physical objects to model problems
- 4.PS.13 Work in collaboration with others to solve problems
- 4.PS.14 Make organized lists to solve numerical problems
- 4.PS.15 Make charts to solve numerical problems
- 4.PS.16 Analyze problems by identifying relationships
- 4.PS.17 Analyze problems by identifying relevant versus irrelevant information
- 4.PS.18 Analyze problems by observing patterns
- 4.PS.19 State a problem in their own words

*Students will monitor and reflect on the process of mathematical problem solving.*

- 4.PS.20 Determine what information is needed to solve a problem
- 4.PS.21 Discuss with peers to understand a problem situation
- 4.PS.22 Discuss the efficiency of different representations of a problem
- 4.PS.23 Verify results of a problem
- 4.PS.24 Recognize invalid approaches
- 4.PS.25 Determine whether a solution is reasonable in the context of the original problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- 4.RP.1 Use representations to support mathematical ideas
- 4.RP.2 Determine whether a mathematical statement is true or false and explain why

*Students will make and investigate mathematical conjectures.*

- 4.RP.3 Investigate the use of knowledgeable guessing by generalizing mathematical ideas
- 4.RP.4 Make conjectures from a variety of representations

*Students will develop and evaluate mathematical arguments and proofs.*

- 4.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions
- 4.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms
- 4.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

*Students will select and use various types of reasoning and methods of proof.*

- 4.RP.8 Support an argument by trying many cases
- 4.RP.9 Disprove an argument by finding counterexamples

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- 4.CM.1 Understand and explain how to organize their thought process
- 4.CM.2 Verbally explain their rationale for strategy selection
- 4.CM.3 Provide reasoning both in written and verbal form

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- 4.CM.4 Organize and accurately label work
- 4.CM.5 Share organized mathematical ideas through the manipulation of objects, drawing, pictures, charts, graphs, tables, diagrams, models, symbols, and expressions in written and verbal form
- 4.CM.6 Answer clarifying questions from others

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- 4.CM.7 Restate mathematical solutions shared by other students
- 4.CM.8 Consider strategies used and solutions found in relation to their own work

*Students will use the language of mathematics to express mathematical ideas precisely.*

- 4.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 4.CM.10 Describe objects, relationships, solutions, and rationale using appropriate vocabulary
- 4.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

### **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- 4.CN.1 Recognize, understand, and make connections in their everyday experiences to mathematical ideas
- 4.CN.2 Compare and contrast mathematical ideas
- 4.CN.3 Connect and apply mathematical information to solve problems

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- 4.CN.4 Understand multiple representations and how they are related
- 4.CN.5 Model situations with objects and representations and be able to make observations

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- 4.CN.6 Recognize the presence of mathematics in their daily lives
- 4.CN.7 Apply mathematics to solve problems that develop outside of mathematics
- 4.CN.8 Recognize and apply mathematics to other disciplines

## Representation Strand

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 4.R.1 Use verbal and written language, physical models, drawing charts, graphs, tables, symbols, and equations as representations
- 4.R.2 Share mental images of mathematical ideas and understandings
- 4.R.3 Recognize and use external mathematical representations
- 4.R.4 Use standard and nonstandard representations with accuracy and detail

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 4.R.5 Understand similarities and differences in representations
- 4.R.6 Connect mathematical representations with problem solving
- 4.R.7 Construct effective representations to solve problems

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 4.R.8 Use mathematics to show and understand physical phenomena (e.g., estimate and represent the number of apples in a tree)
- 4.R.9 Use mathematics to show and understand social phenomena (e.g., determine the number of buses required for a field trip)
- 4.R.10 Use mathematics to show and understand mathematical phenomena (e.g., use a multiplication grid to solve odd and even number problems)

## Number Sense and Operations Strand

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

*Number Systems*      4.N.1 Skip count by 1,000's



- 4.N.2 Read and write whole numbers to 10,000
- 4.N.3 Compare and order numbers to 10,000
- 4.N.4 Understand the place value structure of the base ten number system:  
10 ones = 1 ten  
10 tens = 1 hundred  
10 hundreds = 1 thousand  
10 thousands = 1 ten thousand
- 4.N.5 Recognize equivalent representations for numbers up to four digits and generate them by decomposing and composing numbers
- 4.N.6 Understand, use, and explain the associative property of multiplication
- 4.N.7 Develop an understanding of fractions as locations on number lines and as divisions of whole numbers
- 4.N.8 Recognize and generate equivalent fractions (halves, fourths, thirds, fifths, sixths, and tenths) using manipulatives, visual models, and illustrations
- 4.N.9 Use concrete materials and visual models to compare and order unit fractions or fractions with the same denominator (with and without the use of a number line)
- 4.N.10 Develop an understanding of decimals as part of a whole
- 4.N.11 Read and write decimals to hundredths, using money as a context
- 4.N.12 Use concrete materials and visual models to compare and order decimals (less than 1) to the hundredths place in the context of money
- Number Theory* 4.N.13 Develop an understanding of the properties of odd/even numbers as a result of multiplication

***Students will understand meanings of operations and procedures, and how they relate to one another.***

- Operations* 4.N.14 Use a variety of strategies to add and subtract numbers up to 10,000

- 4.N.15 Select appropriate computational and operational methods to solve problems
- 4.N.16 Understand various meanings of multiplication and division
- 4.N.17 Use multiplication and division as inverse operations to solve problems
- 4.N.18 Use a variety of strategies to multiply two-digit numbers by one-digit numbers (with and without regrouping)
- 4.N.19 Use a variety of strategies to multiply two-digit numbers by two-digit numbers (with and without regrouping)
- 4.N.20 Develop fluency in multiplying and dividing multiples of 10 and 100 up to 1,000
- 4.N.21 Use a variety of strategies to divide two-digit dividends by one-digit divisors (with and without remainders)
- 4.N.22 Interpret the meaning of remainders
- 4.N.23 Add and subtract proper fractions with common denominators
- 4.N.24 Express decimals as an equivalent form of fractions to tenths and hundredths
- 4.N.25 Add and subtract decimals to tenths and hundredths using a hundreds chart

*Students will compute accurately and make reasonable estimates.*

- Estimation* 4.N.26 Round numbers less than 1,000 to the nearest tens and hundreds
- 4.N.27 Check reasonableness of an answer by using estimation

## **Algebra Strand**

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

- Variables and Expressions* 4.A.1 Evaluate and express relationships using open sentences with one operation

*Students will perform algebraic procedures accurately.*

- Equations and Inequalities*
- 4.A.2 Use the symbols  $<$ ,  $>$ ,  $=$ , and  $\neq$  (with and without the use of a number line) to compare whole numbers and unit fractions and decimals (up to hundredths)
  - 4.A.3 Find the value or values that will make an open sentence true, if it contains  $<$  or  $>$

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

- Patterns, Relations, and Functions*
- 4.A.4 Describe, extend, and make generalizations about numeric ( $+$ ,  $-$ ,  $\times$ ,  $\div$ ) and geometric patterns
  - 4.A.5 Analyze a pattern or a whole-number function and state the rule, given a table or an input/output box

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

- Shapes*
- 4.G.1 Identify and name polygons, recognizing that their names are related to the number of sides and angles (triangle, quadrilateral, pentagon, hexagon, and octagon)
  - 4.G.2 Identify points and line segments when drawing a plane figure
  - 4.G.3 Find perimeter of polygons by adding sides
  - 4.G.4 Find the area of a rectangle by counting the number of squares needed to cover the rectangle
  - 4.G.5 Define and identify vertices, faces, and edges of three-dimensional shapes

*Students will identify and justify geometric relationships, formally and informally.*

- Geometric Relationships*
- 4.G.6 Draw and identify intersecting, perpendicular, and parallel lines
  - 4.G.7 Identify points and rays when drawing angles
  - 4.G.8 Classify angles as acute, obtuse, right, and straight

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of  
Measurement*

- 4.M.1 Select tools and units (customary and metric) appropriate for the length being measured
- 4.M.2 Use a ruler to measure to the nearest standard unit (whole,  $\frac{1}{2}$  and  $\frac{1}{4}$  inches, whole feet, whole yards, whole centimeters, and whole meters)
- 4.M.3 Know and understand equivalent standard units of length:  
12 inches = 1 foot  
3 feet = 1 yard
- 4.M.4 Select tools and units appropriate to the mass of the object being measured (grams and kilograms)
- 4.M.5 Measure mass, using grams
- 4.M.6 Select tools and units appropriate to the capacity being measured (milliliters and liters)
- 4.M.7 Measure capacity, using milliliters and liters

*Students will use units to give meaning to measurements.*

*Units*

- 4.M.8 Make change, using combined coins and dollar amounts
- 4.M.9 Calculate elapsed time in hours and half hours, not crossing A.M./P.M.
- 4.M.10 Calculate elapsed time in days and weeks, using a calendar

## Statistics and Probability Strand

*Students will collect, organize, display, and analyze data.*

*Collection of Data*

- 4.S.1 Design investigations to address a question from given data
- 4.S.2 Collect data using observations, surveys, and experiments and record appropriately

*Organization and*

- 4.S.3 Represent data using tables, bar graphs, and pictographs

*Display of Data*

*Analysis of Data*      4.S.4    Read and interpret line graphs

*Students will make predictions that are based upon data analysis.*

*Predictions from Data*      4.S.5    Develop and make predictions that are based on data

4.S.6    Formulate conclusions and make predictions from graphs

## Grade 5

### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

5.PS.1 Know the difference between relevant and irrelevant information when solving problems

5.PS.2 Understand that some ways of representing a problem are more efficient than others

5.PS.3 Interpret information correctly, identify the problem, and generate possible strategies and solutions

*Students will solve problems that arise in mathematics and in other contexts.*

5.PS.4 Act out or model with manipulatives activities involving mathematical content from literature

5.PS.5 Formulate problems and solutions from everyday situations

5.PS.6 Translate from a picture/diagram to a numeric expression

5.PS.7 Represent problem situations verbally, numerically, algebraically, and/or graphically

5.PS.8 Select an appropriate representation of a problem

5.PS.9 Understand the basic language of logic in mathematical situations

(and, or, not)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 5.PS.10 Work in collaboration with others to solve problems
- 5.PS.11 Translate from a picture/diagram to a number or symbolic expression
- 5.PS.12 Use trial and error and the process of elimination to solve problems
- 5.PS.13 Model problems with pictures/diagrams or physical objects
- 5.PS.14 Analyze problems by observing patterns
- 5.PS.15 Make organized lists or charts to solve numerical problems

*Students will monitor and reflect on the process of mathematical problem solving.*

- 5.PS.16 Discuss with peers to understand a problem situation
- 5.PS.17 Determine what information is needed to solve problem
- 5.PS.18 Determine the efficiency of different representations of a problem
- 5.PS.19 Differentiate between valid and invalid approaches
- 5.PS.20 Understand valid counterexamples
- 5.PS.21 Explain the methods and reasoning behind the problem solving strategies used
- 5.PS.22 Discuss whether a solution is reasonable in the context of the original problem
- 5.PS.23 Verify results of a problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- 5.RP.1 Recognize that mathematical ideas can be supported using a variety of strategies

- 5.RP.2 Understand that mathematical statements can be supported, using models, facts, and relationships to explain their thinking

*Students will make and investigate mathematical conjectures.*

- 5.RP.3 Investigate conjectures, using arguments and appropriate mathematical terms

- 5.RP.4 Make and evaluate conjectures, using a variety of strategies

*Students will develop and evaluate mathematical arguments and proofs.*

- 5.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships

- 5.RP.6 Develop and explain an argument verbally, numerically, and/or graphically

- 5.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

*Students will select and use various types of reasoning and methods of proof.*

- 5.RP.8 Support an argument through examples/counterexamples and special cases

## **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- 5.CM.1 Provide an organized thought process that is correct, complete, coherent, and clear

- 5.CM.2 Explain a rationale for strategy selection

- 5.CM.3 Organize and accurately label work

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- 5.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models, and symbols in written and verbal form

5.CM.5 Answer clarifying questions from others

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

5.CM.6 Understand mathematical solutions shared by other students

5.CM.7 Raise questions that elicit, extend, or challenge others' thinking

5.CM.8 Consider strategies used and solutions found by others in relation to their own work

*Students will use the language of mathematics to express mathematical ideas precisely.*

5.CM.9 Increase their use of mathematical vocabulary and language when communicating with others

5.CM.10 Use appropriate vocabulary when describing objects, relationships, mathematical solutions, and rationale

5.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

5.CN.1 Understand and make connections and conjectures in their everyday experiences to mathematical ideas

5.CN.2 Explore and explain the relationship between mathematical ideas

5.CN.3 Connect and apply mathematical information to solve problems

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

5.CN.4 Understand multiple representations and how they are related

5.CN.5 Model situations with objects and representations and be able to draw conclusions

*Students will recognize and apply mathematics in contexts outside of mathematics.*



- 5.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives
- 5.CN.7 Apply mathematics to problem situations that develop outside of mathematics
- 5.CN.8 Investigate the presence of mathematics in careers and areas of interest
- 5.CN.9 Recognize and apply mathematics to other disciplines and areas of interest

### **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 5.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations
- 5.R.2 Explain, describe, and defend mathematical ideas using representations
- 5.R.3 Read, interpret, and extend external models
- 5.R.4 Use standard and nonstandard representations with accuracy and detail

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 5.R.5 Use representations to explore problem situations
- 5.R.6 Investigate relationships between different representations and their impact on a given problem

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 5.R.7 Use mathematics to show and understand physical phenomena (e.g., determine the perimeter of a bulletin board)
- 5.R.8 Use mathematics to show and understand social phenomena (e.g., construct tables to organize data showing book sales)

- 5.R.9 Use mathematics to show and understand mathematical phenomena (e.g., find the missing value that makes the equation true:  $(3 + 4) + 5 = 3 + (4 + \underline{\quad})$ )

## Number Sense and Operations Strand

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

### *Number Systems*

- 5.N.1 Read and write whole numbers to millions
- 5.N.2 Compare and order numbers to millions
- 5.N.3 Understand the place value structure of the base ten number system  
10 ones = 1 ten  
10 tens = 1 hundred  
10 hundreds = 1 thousand  
10 thousands = 1 ten thousand  
10 ten thousands = 1 hundred thousand  
10 hundred thousands = 1 million
- 5.N.4 Create equivalent fractions, given a fraction
- 5.N.5 Compare and order fractions including unlike denominators (with and without the use of a number line) *Note: Commonly used fractions such as those that might be indicated on ruler, measuring cup, etc.*
- 5.N.6 Understand the concept of ratio
- 5.N.7 Express ratios in different forms
- 5.N.8 Read, write, and order decimals to thousandths
- 5.N.9 Compare fractions using  $<$ ,  $>$ , or  $=$
- 5.N.10 Compare decimals using  $<$ ,  $>$ , or  $=$
- 5.N.11 Understand that percent means part of 100, and write percents as fractions and decimals

### *Number Theory*

- 5.N.12 Recognize that some numbers are only divisible by one and themselves (prime) and others have multiple divisors (composite)
- 5.N.13 Calculate multiples of a whole number and the least common

multiple of two numbers

5.N.14 Identify the factors of a given number

5.N.15 Find the common factors and the greatest common factor of two numbers

***Students will understand meanings of operations and procedures, and how they relate to one another.***

*Operations*

5.N.16 Use a variety of strategies to multiply three-digit by three-digit numbers *Note: Multiplication by anything greater than a three-digit multiplier/ multiplicand should be done using technology.*

5.N.17 Use a variety of strategies to divide three-digit numbers by one- and two-digit numbers *Note: Division by anything greater than a two-digit divisor should be done using technology.*

5.N.18 Evaluate an arithmetic expression using order of operations including multiplication, division, addition, subtraction and parentheses

5.N.19 Simplify fractions to lowest terms

5.N.20 Convert improper fractions to mixed numbers, and mixed numbers to improper fractions

5.N.21 Use a variety of strategies to add and subtract fractions with like denominators

5.N.22 Add and subtract mixed numbers with like denominators

5.N.23 Use a variety of strategies to add, subtract, multiply, and divide decimals to thousandths

***Students will compute accurately and make reasonable estimates.***

*Estimation*

5.N.24 Round numbers to the nearest hundredth and up to 10,000

5.N.25 Estimate sums and differences of fractions with like denominators

5.N.26 Estimate sums, differences, products, and quotients of decimals

5.N.27 Justify the reasonableness of answers using estimation

## Algebra Strand

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

- Variables and Expressions*
- 5.A.1 Define and use appropriate terminology when referring to constants, variables, and algebraic expressions
  - 5.A.2 Translate simple verbal expressions into algebraic expressions

*Students will perform algebraic procedures accurately.*

- Variables and Expressions*
- 5.A.3 Substitute assigned values into variable expressions and evaluate using order of operations
- Equations and Inequalities*
- 5.A.4 Solve simple one-step equations using basic whole-number facts
  - 5.A.5 Solve and explain simple one-step equations using inverse operations involving whole numbers
  - 5.A.6 Evaluate the perimeter formula for given input values

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

- Patterns, Relations, and Functions*
- 5.A.7 Create and explain patterns and algebraic relationships (e.g., 2, 4, 6, 8...) algebraically:  $2n$  (doubling)
  - 5.A.8 Create algebraic or geometric patterns using concrete objects or visual drawings (e.g., rotate and shade geometric shapes)

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

- Shapes*
- 5.G.1 Calculate the perimeter of regular and irregular polygons

*Students will identify and justify geometric relationships, formally and informally.*

- Geometric Relationships*
- 5.G.2 Identify pairs of similar triangles
  - 5.G.3 Identify the ratio of corresponding sides of similar triangles
  - 5.G.4 Classify quadrilaterals by properties of their angles and sides
  - 5.G.5 Know that the sum of the interior angles of a quadrilateral is

360 degrees

- 5.G.6 Classify triangles by properties of their angles and sides
- 5.G.7 Know that the sum of the interior angles of a triangle is 180 degrees
- 5.G.8 Find a missing angle when given two angles of a triangle
- 5.G.9 Identify pairs of congruent triangles
- 5.G.10 Identify corresponding parts of congruent triangles

*Students will apply transformations and symmetry to analyze problem solving situations.*

*Transformational Geometry*      5.G.11 Identify and draw lines of symmetry of basic geometric shapes

*Students will apply coordinate geometry to analyze problem solving situations.*

*Coordinate Geometry*      5.G.12 Identify and plot points in the first quadrant

5.G.13 Plot points to form basic geometric shapes (identify and classify)

5.G.14 Calculate perimeter of basic geometric shapes drawn on a coordinate plane (rectangles and shapes composed of rectangles having sides with integer lengths and parallel to the axes)

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of Measurement*      5.M.1 Use a ruler to measure to the nearest inch,  $\frac{1}{2}$ ,  $\frac{1}{4}$ , and  $\frac{1}{8}$  inch

5.M.2 Identify customary equivalent units of length

5.M.3 Measure to the nearest centimeter

5.M.4 Identify equivalent metric units of length

5.M.5 Convert measurement within a given system

*Tools and Methods* 5.M.6 Determine the tool and technique to measure with an appropriate level of precision: lengths and angles

***Students will use units to give meaning to measurements.***

*Units* 5.M.7 Calculate elapsed time in hours and minutes

5.M.8 Measure and draw angles using a protractor

***Students will develop strategies for estimating measurements.***

*Estimation* 5.M.9 Determine personal references for customary units of length (e.g., your pace is approximately 3 feet, your height is approximately 5 feet, etc.)

5.M.10 Determine personal references for metric units of length

5.M.11 Justify the reasonableness of estimates

## **Statistics and Probability Strand**

***Students will collect, organize, display, and analyze data.***

*Collection of Data* 5.S.1 Collect and record data from a variety of sources (e.g., newspapers, magazines, polls, charts, and surveys)

*Organization and Display of Data* 5.S.2 Display data in a line graph to show an increase or decrease over time

*Analysis of Data* 5.S.3 Calculate the mean for a given set of data and use to describe a set of data

***Students will make predictions that are based upon data analysis.***

*Predictions from Data* 5.S.4 Formulate conclusions and make predictions from graphs

***Students will understand and apply concepts of probability.***

*Probability* 5.S.5 List the possible outcomes for a single-event experiment

5.S.6 Record experiment results using fractions/ratios

5.S.7 Create a sample space and determine the probability of a single event, given a simple experiment (e.g., rolling a number cube)

## Grade 6

### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

- 6.PS.1 Know the difference between relevant and irrelevant information when solving problems
- 6.PS.2 Understand that some ways of representing a problem are more efficient than others
- 6.PS.3 Interpret information correctly, identify the problem, and generate possible strategies and solutions

*Students will solve problems that arise in mathematics and in other contexts.*

- 6.PS.4 Act out or model with manipulatives activities involving mathematical content from literature
- 6.PS.5 Formulate problems and solutions from everyday situations
- 6.PS.6 Translate from a picture/diagram to a numeric expression
- 6.PS.7 Represent problem situations verbally, numerically, algebraically, and/or graphically
- 6.PS.8 Select an appropriate representation of a problem
- 6.PS.9 Understand the basic language of logic in mathematical situations (and, or, and not)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 6.PS.10 Work in collaboration with others to solve problems
- 6.PS.11 Translate from a picture/diagram to a number or symbolic expression
- 6.PS.12 Use trial and error and the process of elimination to solve problems

6.PS.13 Model problems with pictures/diagrams or physical objects

6.PS.14 Analyze problems by observing patterns

6.PS.15 Make organized lists or charts to solve numerical problems

*Students will monitor and reflect on the process of mathematical problem solving.*

6.PS.16 Discuss with peers to understand a problem situation

6.PS.17 Determine what information is needed to solve problem

6.PS.18 Determine the efficiency of different representations of a problem

6.PS.19 Differentiate between valid and invalid approaches

6.PS.20 Understand valid counterexamples

6.PS.21 Explain the methods and reasoning behind the problem solving strategies used

6.PS.22 Discuss whether a solution is reasonable in the context of the original problem

6.PS.23 Verify results of a problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

6.RP.1 Recognize that mathematical ideas can be supported using a variety of strategies

6.RP.2 Understand that mathematical statements can be supported, using models, facts, and relationships to explain their thinking

*Students will make and investigate mathematical conjectures.*

6.RP.3 Investigate conjectures, using arguments and appropriate mathematical terms

6.RP.4 Make and evaluate conjectures, using a variety of strategies



*Students will develop and evaluate mathematical arguments and proofs.*

- 6.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships
- 6.RP.6 Develop and explain an argument verbally, numerically, algebraically, and/or graphically
- 6.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

*Students will select and use various types of reasoning and methods of proof.*

- 6.RP.8 Support an argument through examples/counterexamples and special cases
- 6.RP.9 Devise ways to verify results

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- 6.CM.1 Provide an organized thought process that is correct, complete, coherent, and clear
- 6.CM.2 Explain a rationale for strategy selection
- 6.CM.3 Organize and accurately label work

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- 6.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models, and symbols in written and verbal form
- 6.CM.5 Answer clarifying questions from others

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- 6.CM.6 Understand mathematical solutions shared by other students
- 6.CM.7 Raise questions that elicit, extend, or challenge others' thinking

- 6.CM.8 Consider strategies used and solutions found by others in relation to their own work

*Students will use the language of mathematics to express mathematical ideas precisely.*

- 6.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 6.CM.10 Use appropriate vocabulary when describing objects, relationships, mathematical solutions, and rationale
- 6.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

### **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- 6.CN.1 Understand and make connections and conjectures in their everyday experiences to mathematical ideas
- 6.CN.2 Explore and explain the relationship between mathematical ideas
- 6.CN.3 Connect and apply mathematical information to solve problems

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- 6.CN.4 Understand multiple representations and how they are related
- 6.CN.5 Model situations with objects and representations and be able to draw conclusions

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- 6.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives
- 6.CN.7 Apply mathematics to problem situations that develop outside of mathematics
- 6.CN.8 Investigate the presence of mathematics in careers and areas of interest

- 6.CN.9 Recognize and apply mathematics to other disciplines and areas of interest

### **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 6.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations
- 6.R.2 Explain, describe, and defend mathematical ideas using representations
- 6.R.3 Read, interpret, and extend external models
- 6.R.4 Use standard and nonstandard representations with accuracy and detail

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 6.R.5 Use representations to explore problem situations
- 6.R.6 Investigate relationships between different representations and their impact on a given problem

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 6.R.7 Use mathematics to show and understand physical phenomena (e.g., determine the perimeter of a bulletin board)
- 6.R.8 Use mathematics to show and understand social phenomena (e.g., construct tables to organize data showing book sales)
- 6.R.9 Use mathematics to show and understand mathematical phenomena (e.g., Find the missing value:  
 $(3 + 4) + 5 = 3 + (4 + \underline{\quad})$ )

### **Number Sense and Operations Strand**

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

- Number Systems*
- 6.N.1 Read and write whole numbers to trillions
  - 6.N.2 Define and identify the commutative and associative properties of addition and multiplication
  - 6.N.3 Define and identify the distributive property of multiplication over addition
  - 6.N.4 Define and identify the identity and inverse properties of addition and multiplication
  - 6.N.5 Define and identify the zero property of multiplication
  - 6.N.6 Understand the concept of rate
  - 6.N.7 Express equivalent ratios as a proportion
  - 6.N.8 Distinguish the difference between rate and ratio
  - 6.N.9 Solve proportions using equivalent fractions
  - 6.N.10 Verify the proportionality using the product of the means equals the product of the extremes
  - 6.N.11 Read, write, and identify percents of a whole (0% to 100%)
  - 6.N.12 Solve percent problems involving percent, rate, and base
  - 6.N.13 Define absolute value and determine the absolute value of rational numbers (including positive and negative)
  - 6.N.14 Locate rational numbers on a number line (including positive and negative)
  - 6.N.15 Order rational numbers (including positive and negative)

***Students will understand meanings of operations and procedures, and how they relate to one another.***

- Operations*
- 6.N.16 Add and subtract fractions with unlike denominators
  - 6.N.17 Multiply and divide fractions with unlike denominators
  - 6.N.18 Add, subtract, multiply, and divide mixed numbers with unlike denominators

- 6.N.19 Identify the multiplicative inverse (reciprocal) of a number
- 6.N.20 Represent fractions as terminating or repeating decimals
- 6.N.21 Find multiple representations of rational numbers (fractions, decimals, and percents 0 to 100)
- 6.N.22 Evaluate numerical expressions using order of operations (may include exponents of two and three)
- 6.N.23 Represent repeated multiplication in exponential form
- 6.N.24 Represent exponential form as repeated multiplication
- 6.N.25 Evaluate expressions having exponents where the power is an exponent of one, two, or three

*Students will compute accurately and make reasonable estimates.*

*Estimation*

- 6.N.26 Estimate a percent of quantity (0% to 100%)
- 6.N.27 Justify the reasonableness of answers using estimation (including rounding)

## **Algebra Strand**

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

*Variables and Expressions*

- 6.A.1 Translate two-step verbal expressions into algebraic expressions

*Students will perform algebraic procedures accurately.*

*Variables and Expressions*

- 6.A.2 Use substitution to evaluate algebraic expressions (may include exponents of one, two and three)

*Equations and Inequalities*

- 6.A.3 Translate two-step verbal sentences into algebraic equations
- 6.A.4 Solve and explain two-step equations involving whole numbers using inverse operations
- 6.A.5 Solve simple proportions within context

- 6.A.6 Evaluate formulas for given input values (circumference, area, volume, distance, temperature, interest, etc.)

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

### *Shapes*

- 6.G.1 Calculate the length of corresponding sides of similar triangles, using proportional reasoning
- 6.G.2 Determine the area of triangles and quadrilaterals (squares, rectangles, rhombi, and trapezoids) and develop formulas
- 6.G.3 Use a variety of strategies to find the area of regular and irregular polygons
- 6.G.4 Determine the volume of rectangular prisms by counting cubes and develop the formula
- 6.G.5 Identify radius, diameter, chords and central angles of a circle
- 6.G.6 Understand the relationship between the diameter and radius of a circle
- 6.G.7 Determine the area and circumference of a circle, using the appropriate formula
- 6.G.8 Calculate the area of a sector of a circle, given the measure of a central angle and the radius of the circle
- 6.G.9 Understand the relationship between the circumference and the diameter of a circle

*Students will apply coordinate geometry to analyze problem solving situations.*

### *Coordinate Geometry*

- 6.G.10 Identify and plot points in all four quadrants
- 6.G.11 Calculate the area of basic polygons drawn on a coordinate plane (rectangles and shapes composed of rectangles having sides with integer lengths)

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

<i>Units of Measurement</i>	6.M.1	Measure capacity and calculate volume of a rectangular prism
	6.M.2	Identify customary units of capacity (cups, pints, quarts, and gallons)
	6.M.3	Identify equivalent customary units of capacity (cups to pints, pints to quarts, and quarts to gallons)
	6.M.4	Identify metric units of capacity (liter and milliliter)
	6.M.5	Identify equivalent metric units of capacity (milliliter to liter and liter to milliliter)
<i>Tools and Methods</i>	6.M.6	Determine the tool and technique to measure with an appropriate level of precision: capacity

*Students will develop strategies for estimating measurements.*

<i>Estimation</i>	6.M.7	Estimate volume, area, and circumference (see figures identified in geometry strand)
	6.M.8	Justify the reasonableness of estimates
	6.M.9	Determine personal references for capacity

### **Statistics and Probability Strand**

*Students will collect, organize, display, and analyze data.*

<i>Collection of Data</i>	6.S.1	Develop the concept of sampling when collecting data from a population and decide the best method to collect data for a particular question
	6.S.2	Record data in a frequency table
<i>Organization and Display of Data</i>	6.S.3	Construct Venn diagrams to sort data
	6.S.4	Determine and justify the most appropriate graph to display a given set of data (pictograph, bar graph, line graph, histogram, or circle graph)
	6.S.5	Determine the mean, mode and median for a given set of data

6.S.6 Determine the range for a given set of data

6.S.7 Read and interpret graphs

*Students will make predictions that are based upon data analysis.*

*Predictions from Data* 6.S.8 Justify predictions made from data

*Students will understand and apply concepts of probability.*

*Probability* 6.S.9 List possible outcomes for compound events

6.S.10 Determine the probability of dependent events

6.S.11 Determine the number of possible outcomes for a compound event by using the fundamental counting principle and use this to determine the probabilities of events when the outcomes have equal probability

## Grade 7

### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

7.PS.1 Use a variety of strategies to understand new mathematical content and to develop more efficient methods

7.PS.2 Construct appropriate extensions to problem situations

7.PS.3 Understand and demonstrate how written symbols represent mathematical ideas

*Students will solve problems that arise in mathematics and in other contexts.*

7.PS.4 Observe patterns and formulate generalizations

7.PS.5 Make conjectures from generalizations

7.PS.6 Represent problem situations verbally, numerically, algebraically, and graphically



*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 7.PS.7 Understand that there is no one right way to solve mathematical problems but that different methods have advantages and disadvantages
- 7.PS.8 Understand how to break a complex problem into simpler parts or use a similar problem type to solve a problem
- 7.PS.9 Work backwards from a solution
- 7.PS.10 Use proportionality to model problems
- 7.PS.11 Work in collaboration with others to solve problems

*Students will monitor and reflect on the process of mathematical problem solving.*

- 7.PS.12 Interpret solutions within the given constraints of a problem
- 7.PS.13 Set expectations and limits for possible solutions
- 7.PS.14 Determine information required to solve the problem
- 7.PS.15 Choose methods for obtaining required information
- 7.PS.16 Justify solution methods through logical argument
- 7.PS.17 Evaluate the efficiency of different representations of a problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- 7.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

*Students will make and investigate mathematical conjectures.*

- 7.RP.2 Use mathematical strategies to reach a conclusion
- 7.RP.3 Evaluate conjectures by distinguishing relevant from

irrelevant information to reach a conclusion or make appropriate estimates

*Students will develop and evaluate mathematical arguments and proofs.*

7.RP.4 Provide supportive arguments for conjectures

7.RP.5 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

*Students will select and use various types of reasoning and methods of proof.*

7.RP.6 Support an argument by using a systematic approach to test more than one case

7.RP.7 Devise ways to verify results or use counterexamples to refute incorrect statements

7.RP.8 Apply inductive reasoning in making and supporting mathematical conjectures

## **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

7.CM.1 Provide a correct, complete, coherent, and clear rationale for thought process used in problem solving

7.CM.2 Provide an organized argument which explains rationale for strategy selection

7.CM.3 Organize and accurately label work

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

7.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models and symbols in written and verbal form

7.CM.5 Answer clarifying questions from others

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- 7.CM.6 Analyze mathematical solutions shared by others
- 7.CM.7 Compare strategies used and solutions found by others in relation to their own work
- 7.CM.8 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

*Students will use the language of mathematics to express mathematical ideas precisely.*

- 7.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 7.CM.10 Use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale
- 7.CM.11 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- 7.CN.1 Understand and make connections among multiple representations of the same mathematical idea
- 7.CN.2 Recognize connections between subsets of mathematical ideas
- 7.CN.3 Connect and apply a variety of strategies to solve problems

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- 7.CN.4 Model situations mathematically, using representations to draw conclusions and formulate new situations
- 7.CN.5 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- 7.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives
- 7.CN.7 Apply mathematical ideas to problem situations that develop outside of mathematics
- 7.CN.8 Investigate the presence of mathematics in careers and areas of interest
- 7.CN.9 Recognize and apply mathematics to other disciplines, areas of interest, and societal issues

### **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 7.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations
- 7.R.2 Explain, describe, and defend mathematical ideas using representations
- 7.R.3 Recognize, compare, and use an array of representational forms
- 7.R.4 Explain how different representations express the same relationship
- 7.R.5 Use standard and non-standard representations with accuracy and detail

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 7.R.6 Use representations to explore problem situations
- 7.R.7 Investigate relationships between different representations and their impact on a given problem
- 7.R.8 Use representation as a tool for exploring and understanding mathematical ideas

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 7.R.9 Use mathematics to show and understand physical phenomena (e.g., make and interpret scale drawings of figures or scale models of objects)
- 7.R.10 Use mathematics to show and understand social phenomena (e.g., determine profit from sale of yearbooks)
- 7.R.11 Use mathematics to show and understand mathematical phenomena (e.g., use tables, graphs, and equations to show a pattern underlying a function)

### **Number Sense and Operations Strand**

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

#### *Number Systems*

- 7.N.1 Distinguish between the various subsets of real numbers (counting/natural numbers, whole numbers, integers, rational numbers, and irrational numbers)
- 7.N.2 Recognize the difference between rational and irrational numbers (e.g., explore different approximations of  $\pi$ )
- 7.N.3 Place rational and irrational numbers (approximations) on a number line and justify the placement of the numbers
- 7.N.4 Develop the laws of exponents for multiplication and division
- 7.N.5 Write numbers in scientific notation
- 7.N.6 Translate numbers from scientific notation into standard form
- 7.N.7 Compare numbers written in scientific notation

#### *Number Theory*

- 7.N.8 Find the common factors and greatest common factor of two or more numbers
- 7.N.9 Determine multiples and least common multiple of two or more numbers
- 7.N.10 Determine the prime factorization of a given number and write in exponential form

*Students will understand meanings of operations and procedures, and how they relate to one another.*

- Operations*
- 7.N.11 Simplify expressions using order of operations *Note: Expressions may include absolute value and/or integral exponents greater than 0.*
  - 7.N.12 Add, subtract, multiply, and divide integers
  - 7.N.13 Add and subtract two integers (with and without the use of a number line)
  - 7.N.14 Develop a conceptual understanding of negative and zero exponents with a base of ten and relate to fractions and decimals (e.g.,  $10^{-2} = .01 = 1/100$ )
  - 7.N.15 Recognize and state the value of the square root of a perfect square (up to 225)
  - 7.N.16 Determine the square root of non-perfect squares using a calculator
  - 7.N.17 Classify irrational numbers as non-repeating/non-terminating decimals

*Students will compute accurately and make reasonable estimates.*

- Estimation*
- 7.N.18 Identify the two consecutive whole numbers between which the square root of a non-perfect square whole number less than 225 lies (with and without the use of a number line)
  - 7.N.19 Justify the reasonableness of answers using estimation

## **Algebra Strand**

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

- Variables and Expressions*
- 7.A.1 Translate two-step verbal expressions into algebraic expressions

*Students will perform algebraic procedures accurately.*

- Variables and Expressions*
- 7.A.2 Add and subtract monomials with exponents of one
  - 7.A.3 Identify a polynomial as an algebraic expression containing one or more terms

*Equations and Inequalities*

- 7.A.4 Solve multi-step equations by combining like terms, using the distributive property, or moving variables to one side of the equation
- 7.A.5 Solve one-step inequalities (positive coefficients only) (See 7.G.10)
- 7.A.6 Evaluate formulas for given input values (surface area, rate, and density problems)

***Students will recognize, use, and represent algebraically patterns, relations, and functions.***

*Patterns, Relations, and Functions*

- 7.A.7 Draw the graphic representation of a pattern from an equation or from a table of data
- 7.A.8 Create algebraic patterns using charts/tables, graphs, equations, and expressions
- 7.A.9 Build a pattern to develop a rule for determining the sum of the interior angles of polygons
- 7.A.10 Write an equation to represent a function from a table of values

## **Geometry Strand**

***Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.***

*Shapes*

- 7.G.1 Calculate the radius or diameter, given the circumference or area of a circle
- 7.G.2 Calculate the volume of prisms and cylinders, using a given formula and a calculator
- 7.G.3 Identify the two-dimensional shapes that make up the faces and bases of three-dimensional shapes (prisms, cylinders, cones, and pyramids)
- 7.G.4 Determine the surface area of prisms and cylinders, using a calculator and a variety of methods

***Students will identify and justify geometric relationships, formally and informally.***

*Geometric*

- 7.G.5 Identify the right angle, hypotenuse, and legs of a right triangle

*Relationships*

- 7.G.6 Explore the relationship between the lengths of the three sides of a right triangle to develop the Pythagorean Theorem
- 7.G.7 Find a missing angle when given angles of a quadrilateral
- 7.G.8 Use the Pythagorean Theorem to determine the unknown length of a side of a right triangle
- 7.G.9 Determine whether a given triangle is a right triangle by applying the Pythagorean Theorem and using a calculator

*Students will apply coordinate geometry to analyze problem solving situations.*

*Coordinate  
Geometry*

- 7.G.10 Graph the solution set of an inequality (positive coefficients only) on a number line (See 7.A.5)

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of  
Measurement*

- 7.M.1 Calculate distance using a map scale
- 7.M.2 Convert capacities and volumes within a given system
- 7.M.3 Identify customary and metric units of mass
- 7.M.4 Convert mass within a given system
- 7.M.5 Calculate unit price using proportions
- 7.M.6 Compare unit prices
- 7.M.7 Convert money between different currencies with the use of an exchange rate table and a calculator
- 7.M.8 Draw central angles in a given circle using a protractor (circle graphs)

*Tools and Methods*

- 7.M.9 Determine the tool and technique to measure with an appropriate level of precision: mass

*Students will develop strategies for estimating measurements.*



- Estimation*
- 7.M.10 Identify the relationships between relative error and magnitude when dealing with large numbers (e.g., money, population)
  - 7.M.11 Estimate surface area
  - 7.M.12 Determine personal references for customary /metric units of mass
  - 7.M.13 Justify the reasonableness of the mass of an object

### **Statistics and Probability Strand**

*Students will collect, organize, display, and analyze data.*

- Collection of Data*
- 7.S.1 Identify and collect data using a variety of methods
- Organization and Display of Data*
- 7.S.2 Display data in a circle graph
  - 7.S.3 Convert raw data into double bar graphs and double line graphs
- Analysis of Data*
- 7.S.4 Calculate the range for a given set of data
  - 7.S.5 Select the appropriate measure of central tendency
  - 7.S.6 Read and interpret data represented graphically (pictograph, bar graph, histogram, line graph, double line/bar graphs or circle graph)

*Students will make predictions that are based upon data analysis. Predictions from Data* Identify and explain misleading statistics and graphs 7.S.7

*Students will understand and apply concepts of probability.*

- Probability*
- 7.S.8 Interpret data to provide the basis for predictions and to establish experimental probabilities
  - 7.S.9 Determine the validity of sampling methods to predict outcomes
  - 7.S.10 Predict the outcome of an experiment
  - 7.S.11 Design and conduct an experiment to test predictions
  - 7.S.12 Compare actual results to predicted results

## Grade 8

### Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

- 8.PS.1 Use a variety of strategies to understand new mathematical content and to develop more efficient methods
- 8.PS.2 Construct appropriate extensions to problem situations
- 8.PS.3 Understand and demonstrate how written symbols represent mathematical ideas

*Students will solve problems that arise in mathematics and in other contexts.*

- 8.PS.4 Observe patterns and formulate generalizations
- 8.PS.5 Make conjectures from generalizations
- 8.PS.6 Represent problem situations verbally, numerically, algebraically, and graphically

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- 8.PS.7 Understand that there is no one right way to solve mathematical problems but that different methods have advantages and disadvantages
- 8.PS.8 Understand how to break a complex problem into simpler parts or use a similar problem type to solve a problem
- 8.PS.9 Work backwards from a solution
- 8.PS.10 Use proportionality to model problems
- 8.PS.11 Work in collaboration with others to solve problems

*Students will monitor and reflect on the process of mathematical problem solving.*

- 8.PS.12 Interpret solutions within the given constraints of a problem
- 8.PS.13 Set expectations and limits for possible solutions
- 8.PS.14 Determine information required to solve the problem
- 8.PS.15 Choose methods for obtaining required information
- 8.PS.16 Justify solution methods through logical argument
- 8.PS.17 Evaluate the efficiency of different representations of a problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- 8.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

*Students will make and investigate mathematical conjectures.*

- 8.RP.2 Use mathematical strategies to reach a conclusion
- 8.RP.3 Evaluate conjectures by distinguishing relevant from irrelevant information to reach a conclusion or make appropriate estimates

*Students will develop and evaluate mathematical arguments and proofs.*

- 8.RP.4 Provide supportive arguments for conjectures
- 8.RP.5 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

*Students will select and use various types of reasoning and methods of proof.*

- 8.RP.6 Support an argument by using a systematic approach to test more than one case
- 8.RP.7 Devise ways to verify results or use counterexamples to refute incorrect statements
- 8.RP.8 Apply inductive reasoning in making and supporting

mathematical conjectures

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- 8.CM.1 Provide a correct, complete, coherent, and clear rationale for thought process used in problem solving
- 8.CM.2 Provide an organized argument which explains rationale for strategy selection
- 8.CM.3 Organize and accurately label work

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- 8.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models and symbols in written and verbal form
- 8.CM.5 Answer clarifying questions from others

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- 8.CM.6 Analyze mathematical solutions shared by others
- 8.CM.7 Compare strategies used and solutions found by others in relation to their own work
- 8.CM.8 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

*Students will use the language of mathematics to express mathematical ideas precisely.*

- 8.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 8.CM.10 Use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale
- 8.CM.11 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

## Connections Strand

*Students will recognize and use connections among mathematical ideas.*

- 8.CN.1 Understand and make connections among multiple representations of the same mathematical idea
- 8.CN.2 Recognize connections between subsets of mathematical ideas
- 8.CN.3 Connect and apply a variety of strategies to solve problems

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- 8.CN.4 Model situations mathematically, using representations to draw conclusions and formulate new situations
- 8.CN.5 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- 8.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives
- 8.CN.7 Apply mathematical ideas to problem situations that develop outside of mathematics
- 8.CN.8 Investigate the presence of mathematics in careers and areas of interest
- 8.CN.9 Recognize and apply mathematics to other disciplines, areas of interest, and societal issues

## Representation Strand

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- 8.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations

- 8.R.2 Explain, describe, and defend mathematical ideas using representations
- 8.R.3 Recognize, compare, and use an array of representational forms
- 8.R.4 Explain how different representations express the same relationship
- 8.R.5 Use standard and non-standard representations with accuracy and detail

*Students will select, apply, and translate among mathematical representations to solve problems.*

- 8.R.6 Use representations to explore problem situations
- 8.R.7 Investigate relationships between different representations and their impact on a given problem
- 8.R.8 Use representation as a tool for exploring and understanding mathematical ideas

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- 8.R.9 Use mathematics to show and understand physical phenomena (e.g., make and interpret scale drawings of figures or scale models of objects)
- 8.R.10 Use mathematics to show and understand social phenomena (e.g., determine profit from sale of yearbooks)
- 8.R.11 Use mathematics to show and understand mathematical phenomena (e.g., use tables, graphs, and equations to show a pattern underlying a function)

## **Number Sense and Operations Strand**

*Students will understand meanings of operations and procedures, and how they relate to one another.*

*Operations*

- 8.N.1 Develop and apply the laws of exponents for multiplication and division
- 8.N.2 Evaluate expressions with integral exponents

8.N.3 Read, write, and identify percents less than 1% and greater than 100%

8.N.4 Apply percents to:  
Tax  
Percent increase/decrease  
Simple interest  
Sale price  
Commission  
Interest rates  
Gratuities

*Students will compute accurately and make reasonable estimates.*

*Estimation* 8.N.5 Estimate a percent of quantity, given an application

8.N.6 Justify the reasonableness of answers using estimation

### **Algebra Strand**

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

*Variables and Expressions*

8.A.1 Translate verbal sentences into algebraic inequalities

8.A.2 Write verbal expressions that match given mathematical expressions

8.A.3 Describe a situation involving relationships that matches a given graph

8.A.4 Create a graph given a description or an expression for a situation involving a linear or nonlinear relationship

8.A.5 Use physical models to perform operations with polynomials

*Students will perform algebraic procedures accurately.*

*Variables and Expressions*

8.A.6 Multiply and divide monomials

8.A.7 Add and subtract polynomials (integer coefficients)

8.A.8 Multiply a binomial by a monomial or a binomial (integer coefficients)

8.A.9 Divide a polynomial by a monomial (integer coefficients) *Note: The degree of the denominator is less than or equal to the degree of the numerator for all variables.*

8.A.10 Factor algebraic expressions using the GCF

8.A.11 Factor a trinomial in the form  $ax^2 + bx + c$ ;  $a=1$  and  $c$  having no more than three sets of factors

*Equations and  
Inequalities*

8.A.12 Apply algebra to determine the measure of angles formed by or contained in parallel lines cut by a transversal and by intersecting lines

8.A.13 Solve multi-step inequalities and graph the solution set on a number line

8.A.14 Solve linear inequalities by combining like terms, using the distributive property, or moving variables to one side of the inequality (include multiplication or division of inequalities by a negative number)

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

*Patterns, Relations,  
And Functions*

8.A.15 Understand that numerical information can be represented in multiple ways: arithmetically, algebraically, and graphically

8.A.16 Find a set of ordered pairs to satisfy a given linear numerical pattern (expressed algebraically); then plot the ordered pairs and draw the line

8.A.17 Define and use correct terminology when referring to function (domain and range)

8.A.18 Determine if a relation is a function

8.A.19 Interpret multiple representations using equation, table of values, and graph

## Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*



- Constructions*            8.G.0 Construct the following, using a straight edge and compass:  
   Segment congruent to a segment  
   Angle congruent to an angle  
   Perpendicular bisector  
   Angle bisector

***Students will identify and justify geometric relationships, formally and informally.***

- Geometric Relationships*            8.G.1 Identify pairs of vertical angles as congruent
- 8.G.2 Identify pairs of supplementary and complementary angles
- 8.G.3 Calculate the missing angle in a supplementary or complementary pair
- 8.G.4 Determine angle pair relationships when given two parallel lines cut by a transversal
- 8.G.5 Calculate the missing angle measurements when given two parallel lines cut by a transversal
- 8.G.6 Calculate the missing angle measurements when given two intersecting lines and an angle

***Students will apply transformations and symmetry to analyze problem solving situations.***

- Transformational Geometry*            8.G.7 Describe and identify transformations in the plane, using proper function notation (rotations, reflections, translations, and dilations)
- 8.G.8 Draw the image of a figure under rotations of 90 and 180 degrees
- 8.G.9 Draw the image of a figure under a reflection over a given line
- 8.G.10 Draw the image of a figure under a translation
- 8.G.11 Draw the image of a figure under a dilation
- 8.G.12 Identify the properties preserved and not preserved under a reflection, rotation, translation, and dilation

***Students will apply coordinate geometry to analyze problem solving situations.***

- Coordinate Geometry*            8.G.13 Determine the slope of a line from a graph and explain the meaning of slope as a constant rate of change

- 8.G.14 Determine the y-intercept of a line from a graph and be able to explain the y-intercept
- 8.G.15 Graph a line using a table of values
- 8.G.16 Determine the equation of a line given the slope and the y-intercept
- 8.G.17 Graph a line from an equation in slope-intercept form ( $y = mx + b$ )
- 8.G.18 Solve systems of equations graphically (only linear, integral solutions,  $y = mx + b$  format, no vertical/horizontal lines)
- 8.G.19 Graph the solution set of an inequality on a number line
- 8.G.20 Distinguish between linear and nonlinear equations  $ax^2 + bx + c$ ;  $a=1$  (only graphically)
- 8.G.21 Recognize the characteristics of quadratics in tables, graphs, equations, and situations

## Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of  
Measurement*

- 8.M.1 Solve equations/proportions to convert to equivalent measurements within metric and customary measurement systems *Note: Also allow Fahrenheit to Celsius and vice versa.*

## Integrated Algebra

In implementing the Algebra process and content performance indicators, it is expected that students will identify and justify mathematical relationships. The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof, and apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is

therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

**Investigate/Explore** - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

**Discover** - Students will make note of possible patterns and generalizations that result from investigation/exploration.

**Conjecture** - Students will make an overall statement, thought to be true, about the new discovery.

**Reasoning** - Students will engage in a process that leads to knowing something to be true or false.

**Argument** - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

**Justify/Explain** - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

**Proof** - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

**Apply** - Students will use a theorem or concept to solve an algebraic or numerical problem.

## **Problem Solving Strand**

*Students will build new mathematical knowledge through problem solving.*

- A.PS.1 Use a variety of problem solving strategies to understand new mathematical content
- A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

*Students will solve problems that arise in mathematics and in other contexts.*

- A.PS.3 Observe and explain patterns to formulate generalizations and conjectures

- A.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- A.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)
- A.PS.6 Use a variety of strategies to extend solution methods to other problems
- A.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

*Students will monitor and reflect on the process of mathematical problem solving.*

- A.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions
- A.PS.9 Interpret solutions within the given constraints of a problem
- A.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- A.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

*Students will make and investigate mathematical conjectures.*

- A.RP.2 Use mathematical strategies to reach a conclusion and provide supportive arguments for a conjecture
- A.RP.3 Recognize when an approximation is more appropriate than an exact answer

*Students will develop and evaluate mathematical arguments and proofs.*

- A.RP.4 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

- A.RP.5 Construct logical arguments that verify claims or counterexamples that refute them
- A.RP.6 Present correct mathematical arguments in a variety of forms
- A.RP.7 Evaluate written arguments for validity

*Students will select and use various types of reasoning and methods of proof.*

- A.RP.8 Support an argument by using a systematic approach to test more than one case
- A.RP.9 Devise ways to verify results or use counterexamples to refute incorrect statements
- A.RP.10 Extend specific results to more general cases
- A.RP.11 Use a Venn diagram to support a logical argument
- A.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- A.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem
- A.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, Venn diagrams, and other diagrams

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

- A.CM.4 Explain relationships among different representations of a problem
- A.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid
- A.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- A.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students
- A.CM.8 Reflect on strategies of others in relation to one's own strategy
- A.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

*Students will use the language of mathematics to express mathematical ideas precisely.*

- A.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures
- A.CM.11 Represent word problems using standard mathematical notation
- A.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale
- A.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- A.CN.1 Understand and make connections among multiple representations of the same mathematical idea
- A.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
- A.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics
- A.CN.5 Understand how quantitative models connect to various physical models and representations

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- A.CN.6 Recognize and apply mathematics to situations in the outside world
- A.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics
- A.CN.8 Develop an appreciation for the historical development of mathematics

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- A.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
- A.R.2 Recognize, compare, and use an array of representational forms
- A.R.3 Use representation as a tool for exploring and understanding mathematical ideas

*Students will select, apply, and translate among mathematical representations to solve problems.*

- A.R.4 Select appropriate representations to solve problem situations
- A.R.5 Investigate relationships between different representations and their impact on a given problem

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- A.R.6 Use mathematics to show and understand physical phenomena (e.g., find the height of a building if a ladder of a given length forms a given angle of elevation with the ground)
- A.R.7 Use mathematics to show and understand social phenomena (e.g., determine profit from student and adult ticket sales)
- A.R.8 Use mathematics to show and understand mathematical phenomena (e.g., compare the graphs of the functions represented by the equations  $y = x^2$  and  $y = -x^2$ )

### **Number Sense and Operations Strand**

*Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.*

- Number Theory*      A.N.1 Identify and apply the properties of real numbers (closure, commutative, associative, distributive, identity, inverse) *Note: Students do not need to identify groups and fields, but students should be engaged in the ideas.*

*Students will understand meanings of operations and procedures, and how they relate to one another.*

- Operations*      A.N.2 Simplify radical terms (no variable in the radicand)
- A.N.3 Perform the four arithmetic operations using like and unlike radical terms and express the result in simplest form
- A.N.4 Understand and use scientific notation to compute products and quotients of numbers
- A.N.5 Solve algebraic problems arising from situations that involve fractions, decimals, percents (decrease/increase and discount), and proportionality/direct variation
- A.N.6 Evaluate expressions involving factorial(s), absolute value(s), and exponential expression(s)



- A.N.7 Determine the number of possible events, using counting techniques or the Fundamental Principle of Counting
- A.N.8 Determine the number of possible arrangements (permutations) of a list of items

### Algebra Strand

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

*Variables and Expressions*

A.A.1 Translate a quantitative verbal phrase into an algebraic expression

A.A.2 Write a verbal expression that matches a given mathematical expression

*Equations and Inequalities*

A.A.3 Distinguish the difference between an algebraic expression and an algebraic equation

A.A.4 Translate verbal sentences into mathematical equations or inequalities

A.A.5 Write algebraic equations or inequalities that represent a situation

A.A.6 Analyze and solve verbal problems whose solution requires solving a linear equation in one variable or linear inequality in one variable

A.A.7 Analyze and solve verbal problems whose solution requires solving systems of linear equations in two variables

A.A.8 Analyze and solve verbal problems that involve quadratic equations

A.A.9 Analyze and solve verbal problems that involve exponential growth and decay

A.A.10 Solve systems of two linear equations in two variables algebraically (See A.G.7)

A.A.11 Solve a system of one linear and one quadratic equation

in two variables, where only factoring is required *Note: The quadratic equation should represent a parabola and the solution(s) should be integers.*

*Students will perform algebraic procedures accurately.*

*Variables and Expressions*

- A.A.12 Multiply and divide monomial expressions with a common base, using the properties of exponents *Note: Use integral exponents only.*
- A.A.13 Add, subtract, and multiply monomials and polynomials
- A.A.14 Divide a polynomial by a monomial or binomial, where the quotient has no remainder
- A.A.15 Find values of a variable for which an algebraic fraction is undefined
- A.A.16 Simplify fractions with polynomials in the numerator and denominator by factoring both and renaming them to lowest terms
- A.A.17 Add or subtract fractional expressions with monomial or like binomial denominators
- A.A.18 Multiply and divide algebraic fractions and express the product or quotient in simplest form
- A.A.19 Identify and factor the difference of two perfect squares
- A.A.20 Factor algebraic expressions completely, including trinomials with a lead coefficient of one (after factoring a GCF)

*Equations and Inequalities*

- A.A.21 Determine whether a given value is a solution to a given linear equation in one variable or linear inequality in one variable
- A.A.22 Solve all types of linear equations in one variable
- A.A.23 Solve literal equations for a given variable
- A.A.24 Solve linear inequalities in one variable
- A.A.25 Solve equations involving fractional expressions *Note: Expressions which result in linear equations in one variable.*

- A.A.26 Solve algebraic proportions in one variable which result in linear or quadratic equations
- A.A.27 Understand and apply the multiplication property of zero to solve quadratic equations with integral coefficients and integral roots
- A.A.28 Understand the difference and connection between roots of a quadratic equation and factors of a quadratic expression

*Students will recognize, use, and represent algebraically patterns, relations, and functions.*

*Patterns,  
Relations,  
and Functions*

- A.A.29 Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form
- A.A.30 Find the complement of a subset of a given set, within a given universe
- A.A.31 Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets)

*Coordinate  
Geometry*

- A.A.32 Explain slope as a rate of change between dependent and independent variables
- A.A.33 Determine the slope of a line, given the coordinates of two points on the line
- A.A.34 Write the equation of a line, given its slope and the coordinates of a point on the line
- A.A.35 Write the equation of a line, given the coordinates of two points on the line
- A.A.36 Write the equation of a line parallel to the x- or y-axis
- A.A.37 Determine the slope of a line, given its equation in any form
- A.A.38 Determine if two lines are parallel, given their equations in any form
- A.A.39 Determine whether a given point is on a line, given the equation of the line
- A.A.40 Determine whether a given point is in the solution set of a system of linear inequalities

	A.A.41	Determine the vertex and axis of symmetry of a parabola, given its equation (See A.G.10 )
<i>Trigonometric Functions</i>	A.A.42	Find the sine, cosine, and tangent ratios of an angle of a right triangle, given the lengths of the sides
	A.A.43	Determine the measure of an angle of a right triangle, given the length of any two sides of the triangle
	A.A.44	Find the measure of a side of a right triangle, given an acute angle and the length of another side
	A.A.45	Determine the measure of a third side of a right triangle using the Pythagorean theorem, given the lengths of any two sides

### Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

<i>Shapes</i>	A.G.1	Find the area and/or perimeter of figures composed of polygons and circles or sectors of a circle <i>Note: Figures may include triangles, rectangles, squares, parallelograms, rhombuses, trapezoids, circles, semi-circles, quarter-circles, and regular polygons (perimeter only).</i>
	A.G.2	Use formulas to calculate volume and surface area of rectangular solids and cylinders

*Students will apply coordinate geometry to analyze problem solving situations.*

<i>Coordinate Geometry</i>	A.G.3	Determine when a relation is a function, by examining ordered pairs and inspecting graphs of relations
	A.G.4	Identify and graph linear, quadratic (parabolic), absolute value, and exponential functions
	A.G.5	Investigate and generalize how changing the coefficients of a function affects its graph
	A.G.6	Graph linear inequalities
	A.G.7	Graph and solve systems of linear equations and inequalities with rational coefficients in two variables (See A.A.10)

- A.G.8 Find the roots of a parabolic function graphically *Note: Only quadratic equations with integral solutions.*
- A.G.9 Solve systems of linear and quadratic equations graphically *Note: Only use systems of linear and quadratic equations that lead to solutions whose coordinates are integers.*
- A.G.10 Determine the vertex and axis of symmetry of a parabola, given its graph (See A.A.41) *Note: The vertex will have an ordered pair of integers and the axis of symmetry will have an integral value.*

### Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of Measurement*

- A.M.1 Calculate rates using appropriate units (e.g., rate of a space ship versus the rate of a snail)
- A.M.2 Solve problems involving conversions within measurement systems, given the relationship between the units

*Students will understand that all measurement contains error and be able to determine its significance.*

*Error and Magnitude*

- A.M.3 Calculate the relative error in measuring square and cubic units, when there is an error in the linear measure

### Statistics and Probability Strand

*Students will collect, organize, display, and analyze data.*

*Organization and Display of Data*

- A.S.1 Categorize data as qualitative or quantitative
- A.S.2 Determine whether the data to be analyzed is univariate or bivariate
- A.S.3 Determine when collected data or display of data may be biased
- A.S.4 Compare and contrast the appropriateness of different measures of central tendency for a given data set

- A.S.5 Construct a histogram, cumulative frequency histogram, and a box-and-whisker plot, given a set of data
- A.S.6 Understand how the five statistical summary (minimum, maximum, and the three quartiles) is used to construct a box-and-whisker plot
- A.S.7 Create a scatter plot of bivariate data
- A.S.8 Construct manually a reasonable line of best fit for a scatter plot and determine the equation of that line
- Analysis of Data* A.S.9 Analyze and interpret a frequency distribution table or histogram, a cumulative frequency distribution table or histogram, or a box-and-whisker plot
- A.S.10 Evaluate published reports and graphs that are based on data by considering: experimental design, appropriateness of the data analysis, and the soundness of the conclusions
- A.S.11 Find the percentile rank of an item in a data set and identify the point values for first, second, and third quartiles
- A.S.12 Identify the relationship between the independent and dependent variables from a scatter plot (positive, negative, or none)
- A.S.13 Understand the difference between correlation and causation
- A.S.14 Identify variables that might have a correlation but not a causal relationship

***Students will make predictions that are based upon data analysis.***

- Predictions from Data* A.S.15 Identify and describe sources of bias and its effect, drawing conclusions from data
- A.S.16 Recognize how linear transformations of one-variable data affect the data's mean, median, mode, and range
- A.S.17 Use a reasonable line of best fit to make a prediction involving interpolation or extrapolation

***Students will understand and apply concepts of probability.***

- Probability*
- A.S.18 Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces
  - A.S.19 Determine the number of elements in a sample space and the number of favorable events
  - A.S.20 Calculate the probability of an event and its complement
  - A.S.21 Determine empirical probabilities based on specific sample data
  - A.S.22 Determine, based on calculated probability of a set of events, if:
    - some or all are equally likely to occur
    - one is more likely to occur than another
    - whether or not an event is certain to happen or not to happen
  - A.S.23 Calculate the probability of:
    - a series of independent events
    - a series of dependent events
    - two mutually exclusive events
    - two events that are not mutually exclusive

## Geometry

In implementing the Geometry process and content performance indicators, it is expected that students will identify and justify geometric relationships, formally and informally. For example, students will begin with a definition of a figure and from that definition students will be expected to develop a list of conjectured properties of the figure and to justify each conjecture informally or with formal proof. Students will also be expected to list the assumptions that are needed in order to justify each conjectured property and present their findings in an organized manner.

The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. The variety of approaches to verification and proof is what gives curriculum developers and

teachers the flexibility to adapt strategies to address these performance indicators in a manner that meets the diverse needs of our students. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof, and apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

**Investigate/Explore** - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

**Discover** - Students will make note of possible relationships of perpendicularity, parallelism, congruence, and/or similarity after investigation/exploration.

**Conjecture** - Students will make an overall statement, thought to be true, about the new discovery.

**Reasoning** - Students will engage in a process that leads to knowing something to be true or false.

**Argument** - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

**Justify/Explain** - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

**Proof** - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems using properties of perpendicularity, parallelism, congruence, and similarity with polygons and circles.

**Apply** - Students will use a theorem or concept to solve a geometric problem.

## Problem Solving Strand

*Students will build new mathematical knowledge through problem solving.*

G.PS.1 Use a variety of problem solving strategies to understand new mathematical content

*Students will solve problems that arise in mathematics and in other contexts.*



- G.PS.2 Observe and explain patterns to formulate generalizations and conjectures
- G.PS.3 Use multiple representations to represent and explain problem situations (e.g., spatial, geometric, verbal, numeric, algebraic, and graphical representations)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- G.PS.4 Construct various types of reasoning, arguments, justifications and methods of proof for problems
- G.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)
- G.PS.6 Use a variety of strategies to extend solution methods to other problems
- G.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

*Students will monitor and reflect on the process of mathematical problem solving.*

- G.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions
- G.PS.9 Interpret solutions within the given constraints of a problem
- G.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

### **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

- G.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies
- G.RP.2 Recognize and verify, where appropriate, geometric relationships of perpendicularity, parallelism, congruence, and similarity, using algebraic strategies

*Students will make and investigate mathematical conjectures.*

- G.RP.3 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

*Students will develop and evaluate mathematical arguments and proofs.*

- G.RP.4 Provide correct mathematical arguments in response to other students' conjectures, reasoning, and arguments
- G.RP.5 Present correct mathematical arguments in a variety of forms
- G.RP.6 Evaluate written arguments for validity

*Students will select and use various types of reasoning and methods of proof.*

- G.RP.7 Construct a proof using a variety of methods (e.g., deductive, analytic, transformational)
- G.RP.8 Devise ways to verify results or use counterexamples to refute incorrect statements
- G.RP.9 Apply inductive reasoning in making and supporting mathematical conjectures

### **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

- G.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem
- G.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- G.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form
- G.CM.4 Explain relationships among different representations of a problem

- G.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid
- G.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- G.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students
- G.CM.8 Reflect on strategies of others in relation to one's own strategy
- G.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

*Students will use the language of mathematics to express mathematical ideas precisely.*

- G.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures
- G.CM.11 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and geometric diagrams
- G.CM.12 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- G.CN.1 Understand and make connections among multiple representations of the same mathematical idea
- G.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- G.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
- G.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics
- G.CN.5 Understand how quantitative models connect to various physical models and representations

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- G.CN.6 Recognize and apply mathematics to situations in the outside world
- G.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics
- G.CN.8 Develop an appreciation for the historical development of mathematics

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*

- G.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
- G.R.2 Recognize, compare, and use an array of representational forms
- G.R.3 Use representation as a tool for exploring and understanding mathematical ideas

*Students will select, apply, and translate among mathematical representations to solve problems.*

- G.R.4 Select appropriate representations to solve problem situations
- G.R.5 Investigate relationships between different representations and their impact on a given problem

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- G.R.6 Use mathematics to show and understand physical phenomena (e.g., determine the number of gallons of water in a fish tank)
- G.R.7 Use mathematics to show and understand social phenomena (e.g., determine if conclusions from another person's argument have a logical foundation)
- G.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use investigation, discovery, conjecture, reasoning, arguments, justification and proofs to validate that the two base angles of an isosceles triangle are congruent)

### Algebra Strand

*Note: The algebraic skills and concepts within the Algebra process and content performance indicators must be maintained and applied as students are asked to investigate, make conjectures, give rationale, and justify or prove geometric concepts.*

### Geometry Strand

*Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.*

#### *Geometric Relationships*

*Note: Two-dimensional geometric relationships are addressed in the Informal and Formal Proofs band.*

- G.G.1 Know and apply that if a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them
- G.G.2 Know and apply that through a given point there passes one and only one plane perpendicular to a given line
- G.G.3 Know and apply that through a given point there passes one and only one line perpendicular to a given plane
- G.G.4 Know and apply that two lines perpendicular to the same plane are coplanar
- G.G.5 Know and apply that two planes are perpendicular to each other if and only if one plane contains a line perpendicular to the second plane

- G.G.6 Know and apply that if a line is perpendicular to a plane, then any line perpendicular to the given line at its point of intersection with the given plane is in the given plane
- G.G.7 Know and apply that if a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane
- G.G.8 Know and apply that if a plane intersects two parallel planes, then the intersection is two parallel lines
- G.G.9 Know and apply that if two planes are perpendicular to the same line, they are parallel
- G.G.10 Know and apply that the lateral edges of a prism are congruent and parallel
- G.G.11 Know and apply that two prisms have equal volumes if their bases have equal areas and their altitudes are equal
- G.G.12 Know and apply that the volume of a prism is the product of the area of the base and the altitude
- G.G.13 Apply the properties of a regular pyramid, including:
- lateral edges are congruent
  - lateral faces are congruent isosceles triangles
  - volume of a pyramid equals one-third the product of the area of the base and the altitude
- G.G.14 Apply the properties of a cylinder, including:
- bases are congruent
  - volume equals the product of the area of the base and the altitude
  - lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base
- G.G.15 Apply the properties of a right circular cone, including:
- lateral area equals one-half the product of the slant height and the circumference of its base
  - volume is one-third the product of the area of its base and its altitude
- G.G.16 Apply the properties of a sphere, including:
- the intersection of a plane and a sphere is a circle

- a great circle is the largest circle that can be drawn on a sphere
- two planes equidistant from the center of the sphere and intersecting the sphere do so in congruent circles
- surface area is  $4\pi r^2$
- volume is  $\frac{4}{3}\pi r^3$

*Constructions*

- G.G.17 Construct a bisector of a given angle, using a straightedge and compass, and justify the construction
- G.G.18 Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction
- G.G.19 Construct lines parallel (or perpendicular) to a given line through a given point, using a straightedge and compass, and justify the construction
- G.G.20 Construct an equilateral triangle, using a straightedge and compass, and justify the construction

*Locus*

- G.G.21 Investigate and apply the concurrence of medians, altitudes, angle bisectors, and perpendicular bisectors of triangles
- G.G.22 Solve problems using compound loci
- G.G.23 Graph and solve compound loci in the coordinate plane

***Students will identify and justify geometric relationships formally and informally.***

*Informal and  
Formal Proofs*

- G.G.24 Determine the negation of a statement and establish its truth value
- G.G.25 Know and apply the conditions under which a compound statement (conjunction, disjunction, conditional, biconditional) is true
- G.G.26 Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences
- G.G.27 Write a proof arguing from a given hypothesis to a given conclusion
- G.G.28 Determine the congruence of two triangles by using one of the five congruence techniques (SSS, SAS, ASA, AAS, HL), given

sufficient information about the sides and/or angles of two congruent triangles

- G.G.29 Identify corresponding parts of congruent triangles
- G.G.30 Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle
- G.G.31 Investigate, justify, and apply the isosceles triangle theorem and its converse
- G.G.32 Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem
- G.G.33 Investigate, justify, and apply the triangle inequality theorem
- G.G.34 Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle
- G.G.35 Determine if two lines cut by a transversal are parallel, based on the measure of given pairs of angles formed by the transversal and the lines
- G.G.36 Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons
- G.G.37 Investigate, justify, and apply theorems about each interior and exterior angle measure of regular polygons
- G.G.38 Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals
- G.G.39 Investigate, justify, and apply theorems about special parallelograms (rectangles, rhombuses, squares) involving their angles, sides, and diagonals
- G.G.40 Investigate, justify, and apply theorems about trapezoids (including isosceles trapezoids) involving their angles, sides, medians, and diagonals
- G.G.41 Justify that some quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids



- G.G.42 Investigate, justify, and apply theorems about geometric relationships, based on the properties of the line segment joining the midpoints of two sides of the triangle
- G.G.43 Investigate, justify, and apply theorems about the centroid of a triangle, dividing each median into segments whose lengths are in the ratio 2:1
- G.G.44 Establish similarity of triangles, using the following theorems: AA, SAS, and SSS
- G.G.45 Investigate, justify, and apply theorems about similar triangles
- G.G.46 Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle
- G.G.47 Investigate, justify, and apply theorems about mean proportionality:
- the altitude to the hypotenuse of a right triangle is the mean proportional between the two segments along the hypotenuse
  - the altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg of the right triangle is the mean proportional between the hypotenuse and segment of the hypotenuse adjacent to that leg
- G.G.48 Investigate, justify, and apply the Pythagorean theorem and its converse
- G.G.49 Investigate, justify, and apply theorems regarding chords of a circle:
- perpendicular bisectors of chords
  - the relative lengths of chords as compared to their distance from the center of the circle
- G.G.50 Investigate, justify, and apply theorems about tangent lines to a circle:
- a perpendicular to the tangent at the point of tangency
  - two tangents to a circle from the same external point
  - common tangents of two non-intersecting or tangent circles

- G.G.51 Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle when the vertex is:
- inside the circle (two chords)
  - on the circle (tangent and chord)
  - outside the circle (two tangents, two secants, or tangent and secant)
- G.G.52 Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines
- G.G.53 Investigate, justify, and apply theorems regarding segments intersected by a circle:
- along two tangents from the same external point
  - along two secants from the same external point
  - along a tangent and a secant from the same external point
  - along two intersecting chords of a given circle

***Students will apply transformations and symmetry to analyze problem solving situations.***

*Transformational  
Geometry*

- G.G.54 Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections)  
*Note: Use proper function notation.*
- G.G.55 Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections
- G.G.56 Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism
- G.G.57 Justify geometric relationships (perpendicularity, parallelism, congruence) using transformational techniques (translations, rotations, reflections)
- G.G.58 Define, investigate, justify, and apply similarities (dilations and the composition of dilations and isometries)
- G.G.59 Investigate, justify, and apply the properties that remain invariant under similarities
- G.G.60 Identify specific similarities by observing orientation, numbers of invariant points, and/or parallelism

- G.G.61 Investigate, justify, and apply the analytical representations for translations, rotations about the origin of  $90^\circ$  and  $180^\circ$ , reflections over the lines  $x = 0$ ,  $y = 0$ , and  $y = x$ , and dilations centered at the origin

*Students will apply coordinate geometry to analyze problem solving situations.*

*Coordinate  
Geometry*

- G.G.62 Find the slope of a perpendicular line, given the equation of a line
- G.G.63 Determine whether two lines are parallel, perpendicular, or neither, given their equations
- G.G.64 Find the equation of a line, given a point on the line and the equation of a line perpendicular to the given line
- G.G.65 Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line
- G.G.66 Find the midpoint of a line segment, given its endpoints
- G.G.67 Find the length of a line segment, given its endpoints
- G.G.68 Find the equation of a line that is the perpendicular bisector of a line segment, given the endpoints of the line segment
- G.G.69 Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas
- G.G.70 Solve systems of equations involving one linear equation and one quadratic equation graphically
- G.G.71 Write the equation of a circle, given its center and radius or given the endpoints of a diameter
- G.G.72 Write the equation of a circle, given its graph  
*Note: The center is an ordered pair of integers and the radius is an integer.*
- G.G.73 Find the center and radius of a circle, given the equation of the circle in center-radius form
- G.G.74 Graph circles of the form  $(x - h)^2 + (y - k)^2 = r^2$

## Algebra 2 and Trigonometry

In implementing the Algebra 2 and Trigonometry process and content performance indicators, it is expected that students will identify and justify mathematical relationships, formally and informally. The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate*, *explore*, *discover*, *conjecture*, *reasoning*, *argument*, *justify*, *explain*, *proof*, and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

**Investigate/Explore** - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

**Discover** - Students will make note of possible patterns and generalizations that result from investigation/exploration.

**Conjecture** - Students will make an overall statement, thought to be true, about the new discovery.

**Reasoning** - Students will engage in a process that leads to knowing something to be true or false.

**Argument** - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

**Justify/Explain** - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

**Proof** - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

**Apply** - Students will use a theorem or concept to solve an algebraic or numerical problem.

## **Problem Solving Strand**

*Students will build new mathematical knowledge through problem solving.*

- A2.PS.1 Use a variety of problem solving strategies to understand new mathematical content
- A2.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

*Students will solve problems that arise in mathematics and in other contexts.*

- A2.PS.3 Observe and explain patterns to formulate generalizations and conjectures
- A2.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

*Students will apply and adapt a variety of appropriate strategies to solve problems.*

- A2.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)
- A2.PS.6 Use a variety of strategies to extend solution methods to other problems
- A2.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

*Students will monitor and reflect on the process of mathematical problem solving.*

- A2.PS.8 Determine information required to solve the problem, choose methods for obtaining the information, and define parameters for acceptable solutions
- A2.PS.9 Interpret solutions within the given constraints of a problem
- A2.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

## **Reasoning and Proof Strand**

*Students will recognize reasoning and proof as fundamental aspects of mathematics.*

A2.RP.1 Support mathematical ideas using a variety of strategies

*Students will make and investigate mathematical conjectures.*

A2.RP.2 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

A2.RP.3 Evaluate conjectures and recognize when an estimate or approximation is more appropriate than an exact answer

A2.RP.4 Recognize when an approximation is more appropriate than an exact answer

*Students will develop and evaluate mathematical arguments and proofs.*

A2.RP.5 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

A2.RP.6 Construct logical arguments that verify claims or counterexamples that refute claims

A2.RP.7 Present correct mathematical arguments in a variety of forms

A2.RP.8 Evaluate written arguments for validity

*Students will select and use various types of reasoning and methods of proof.*

A2.RP.9 Support an argument by using a systematic approach to test more than one case

A2.RP.10 Devise ways to verify results, using counterexamples and informal indirect proof

A2.RP.11 Extend specific results to more general cases

A2.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

## **Communication Strand**

*Students will organize and consolidate their mathematical thinking through communication.*

A2.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

- A2.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

*Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.*

- A2.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form
- A2.CM.4 Explain relationships among different representations of a problem
- A2.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid
- A2.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

*Students will analyze and evaluate the mathematical thinking and strategies of others.*

- A2.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students
- A2.CM.8 Reflect on strategies of others in relation to one's own strategy
- A2.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

*Students will use the language of mathematics to express mathematical ideas precisely.*

- A2.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures
- A2.CM.11 Represent word problems using standard mathematical notation
- A2.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale
- A2.CM.13 Draw conclusions about mathematical ideas through

decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

## **Connections Strand**

*Students will recognize and use connections among mathematical ideas.*

- A2.CN.1 Understand and make connections among multiple representations of the same mathematical idea
- A2.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

*Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.*

- A2.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
- A2.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics
- A2.CN.5 Understand how quantitative models connect to various physical models and representations

*Students will recognize and apply mathematics in contexts outside of mathematics.*

- A2.CN.6 Recognize and apply mathematics to situations in the outside world
- A2.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics
- A2.CN.8 Develop an appreciation for the historical development of mathematics

## **Representation Strand**

*Students will create and use representations to organize, record, and communicate mathematical ideas.*



- A2.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts
- A2.R.2 Recognize, compare, and use an array of representational forms
- A2.R.3 Use representation as a tool for exploring and understanding mathematical ideas

*Students will select, apply, and translate among mathematical representations to solve problems.*

- A2.R.4 Select appropriate representations to solve problem situations
- A2.R.5 Investigate relationships among different representations and their impact on a given problem

*Students will use representations to model and interpret physical, social, and mathematical phenomena.*

- A2.R.6 Use mathematics to show and understand physical phenomena (e.g., investigate sound waves using the sine and cosine functions)
- A2.R.7 Use mathematics to show and understand social phenomena (e.g., interpret the results of an opinion poll)
- A2.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use random number generator to simulate a coin toss)

## **Number Sense and Operations Strand**

*Students will understand meanings of operations and procedures, and how they relate to one another.*

- Operations* A2.N.1 Evaluate numerical expressions with negative and/or fractional exponents, without the aid of a calculator (when the answers are rational numbers)
- A2.N.2 Perform arithmetic operations (addition, subtraction, multiplication, division) with expressions containing irrational numbers in radical form

- A2.N.3 Perform arithmetic operations with polynomial expressions containing rational coefficients
- A2.N.4 Perform arithmetic operations on irrational expressions
- A2.N.5 Rationalize a denominator containing a radical expression
- A2.N.6 Write square roots of negative numbers in terms of  $i$
- A2.N.7 Simplify powers of  $i$
- A2.N.8 Determine the conjugate of a complex number
- A2.N.9 Perform arithmetic operations on complex numbers and write the answer in the form  $a + bi$  *Note: This includes simplifying expressions with complex denominators.*
- A2.N.10 Know and apply sigma notation

### Algebra Strand

*Students will represent and analyze algebraically a wide variety of problem solving situations.*

*Equations and Inequalities*

- A2.A.1 Solve absolute value equations and inequalities involving linear expressions in one variable
- A2.A.2 Use the discriminant to determine the nature of the roots of a quadratic equation
- A2.A.3 Solve systems of equations involving one linear equation and one quadratic equation algebraically *Note: This includes rational equations that result in linear equations with extraneous roots.*
- A2.A.4 Solve quadratic inequalities in one and two variables, algebraically and graphically
- A2.A.5 Use direct and inverse variation to solve for unknown values
- A2.A.6 Solve an application which results in an exponential function

*Students will perform algebraic procedures accurately.*

*Variables and Expressions*

- A2.A.7 Factor polynomial expressions completely, using any combination of the following techniques: common factor extraction, difference of two perfect squares, quadratic trinomials
- A2.A.8 Apply the rules of exponents to simplify expressions involving negative and/or fractional exponents
- A2.A.9 Rewrite algebraic expressions that contain negative exponents using only positive exponents
- A2.A.10 Rewrite algebraic expressions with fractional exponents as radical expressions
- A2.A.11 Rewrite algebraic expressions in radical form as expressions with fractional exponents
- A2.A.12 Evaluate exponential expressions, including those with base  $e$
- A2.A.13 Simplify radical expressions
- A2.A.14 Perform addition, subtraction, multiplication, and division of radical expressions
- A2.A.15 Rationalize denominators involving algebraic radical expressions
- A2.A.16 Perform arithmetic operations with rational expressions and rename to lowest terms
- A2.A.17 Simplify complex fractional expressions
- A2.A.18 Evaluate logarithmic expressions in any base
- A2.A.19 Apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms

*Equations and Inequalities*

- A2.A.20 Determine the sum and product of the roots of a quadratic equation by examining its coefficients
- A2.A.21 Determine the quadratic equation, given the sum and product of its roots
- A2.A.22 Solve radical equations
- A2.A.23 Solve rational equations and inequalities

- A2.A.24 Know and apply the technique of completing the square
- A2.A.25 Solve quadratic equations, using the quadratic formula
- A2.A.26 Find the solution to polynomial equations of higher degree that can be solved using factoring and/or the quadratic formula
- A2.A.27 Solve exponential equations with and without common bases
- A2.A.28 Solve a logarithmic equation by rewriting as an exponential equation

***Students will recognize, use, and represent algebraically patterns, relations, and functions.***

*Patterns,  
Relations,  
and Functions*

- A2.A.29 Identify an arithmetic or geometric sequence and find the formula for its  $n$ th term
- A2.A.30 Determine the common difference in an arithmetic sequence
- A2.A.31 Determine the common ratio in a geometric sequence
- A2.A.32 Determine a specified term of an arithmetic or geometric sequence
- A2.A.33 Specify terms of a sequence, given its recursive definition
- A2.A.34 Represent the sum of a series, using sigma notation
- A2.A.35 Determine the sum of the first  $n$  terms of an arithmetic or geometric series
- A2.A.36 Apply the binomial theorem to expand a binomial and determine a specific term of a binomial expansion
- A2.A.37 Define a relation and function
- A2.A.38 Determine when a relation is a function
- A2.A.39 Determine the domain and range of a function from its equation
- A2.A.40 Write functions in functional notation
- A2.A.41 Use functional notation to evaluate functions for given values in the domain

- A2.A.42 Find the composition of functions
- A2.A.43 Determine if a function is one-to-one, onto, or both
- A2.A.44 Define the inverse of a function
- A2.A.45 Determine the inverse of a function and use composition to justify the result
- A2.A.46 Perform transformations with functions and relations:  
 $f(x+a)$ ,  $f(x)+a$ ,  $f(-x)$ ,  $-f(x)$ ,  $af(x)$

*Coordinate  
Geometry*

- A2.A.47 Determine the center-radius form for the equation of a circle in standard form
- A2.A.48 Write the equation of a circle, given its center and a point on the circle
- A2.A.49 Write the equation of a circle from its graph
- A2.A.50 Approximate the solution to polynomial equations of higher degree by inspecting the graph
- A2.A.51 Determine the domain and range of a function from its graph
- A2.A.52 Identify relations and functions, using graphs
- A2.A.53 Graph exponential functions of the form  $y = b^x$  for positive values of  $b$ , including  $b = e$
- A2.A.54 Graph logarithmic functions, using the inverse of the related exponential function

*Trigonometric  
Functions*

- A2.A.55 Express and apply the six trigonometric functions as ratios of the sides of a right triangle
- A2.A.56 Know the exact and approximate values of the sine, cosine, and tangent of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$  angles
- A2.A.57 Sketch and use the reference angle for angles in standard position
- A2.A.58 Know and apply the co-function and reciprocal relationships between trigonometric ratios

- A2.A.59 Use the reciprocal and co-function relationships to find the value of the secant, cosecant, and cotangent of  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$  angles
- A2.A.60 Sketch the unit circle and represent angles in standard position
- A2.A.61 Determine the length of an arc of a circle, given its radius and the measure of its central angle
- A2.A.62 Find the value of trigonometric functions, if given a point on the terminal side of angle  $\theta$
- A2.A.63 Restrict the domain of the sine, cosine, and tangent functions to ensure the existence of an inverse function
- A2.A.64 Use inverse functions to find the measure of an angle, given its sine, cosine, or tangent
- A2.A.65 Sketch the graph of the inverses of the sine, cosine, and tangent functions
- A2.A.66 Determine the trigonometric functions of any angle, using technology
- A2.A.67 Justify the Pythagorean identities
- A2.A.68 Solve trigonometric equations for all values of the variable from  $0^\circ$  to  $360^\circ$
- A2.A.69 Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function
- A2.A.70 Sketch and recognize one cycle of a function of the form  $y = A \sin Bx$  or  $y = A \cos Bx$
- A2.A.71 Sketch and recognize the graphs of the functions  $y = \sec(x)$ ,  $y = \csc(x)$ ,  $y = \tan(x)$ , and  $y = \cot(x)$
- A2.A.72 Write the trigonometric function that is represented by a given periodic graph
- A2.A.73 Solve for an unknown side or angle, using the Law of Sines or the Law of Cosines
- A2.A.74 Determine the area of a triangle or a parallelogram, given the measure of two sides and the included angle

- A2.A.75 Determine the solution(s) from the SSA situation (ambiguous case)
- A2.A.76 Apply the angle sum and difference formulas for trigonometric functions
- A2.A.77 Apply the double-angle and half-angle formulas for trigonometric functions

### Measurement Strand

*Students will determine what can be measured and how, using appropriate methods and formulas.*

*Units of  
Measurement*

- A2.M.1 Define radian measure
- A2.M.2 Convert between radian and degree measures

### Statistics and Probability Strand

*Students will collect, organize, display, and analyze data.*

*Collection of  
Data*

- A2.S.1 Understand the differences among various kinds of studies (e.g., survey, observation, controlled experiment)
- A2.S.2 Determine factors which may affect the outcome of a survey

*Organization and  
Display of Data*

- A2.S.3 Calculate measures of central tendency with group frequency distributions
- A2.S.4 Calculate measures of dispersion (range, quartiles, interquartile range, standard deviation, variance) for both samples and populations
- A2.S.5 Know and apply the characteristics of the normal distribution

*Students will make predictions that are based upon data analysis.*

*Predictions from  
Data*

- A2.S.6 Determine from a scatter plot whether a linear, logarithmic, exponential, or power regression model is most appropriate

- A2.S.7 Determine the function for the regression model, using appropriate technology, and use the regression function to interpolate and extrapolate from the data
- A2.S.8 Interpret within the linear regression model the value of the correlation coefficient as a measure of the strength of the relationship

*Students will understand and apply concepts of probability.*

*Probability*

- A2.S.9 Differentiate between situations requiring permutations and those requiring combinations
- A2.S.10 Calculate the number of possible permutations ( ${}_n P_r$ ) of  $n$  items taken  $r$  at a time
- A2.S.11 Calculate the number of possible combinations ( ${}_n C_r$ ) of  $n$  items taken  $r$  at a time
- A2.S.12 Use permutations, combinations, and the Fundamental Principle of Counting to determine the number of elements in a sample space and a specific subset (event)
- A2.S.13 Calculate theoretical probabilities, including geometric applications
- A2.S.14 Calculate empirical probabilities
- A2.S.15 Know and apply the binomial probability formula to events involving the terms *exactly*, *at least*, and *at most*
- A2.S.16 Use the normal distribution as an approximation for binomial probabilities